Quant Market Econ (2007) 5:131–161
DOI 10.1007/s11129-006-9018-9

Advertising budgets in competitive environments

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Received: 12 November 2005 / Accepted: 21 December 2006 /
Published online: 2 June 2007

Abstract Firms can approach advertising competition either by setting advertising budgets (as in the percentage of sales method) or target sales levels (as in the objective and task approach). We study firms' incentives to adopt one or the other posture using a two-stage model of duopolistic competition. In the first stage, each firm chooses to commit either to an advertising budget, letting its sales follow from the market response function, or to a desired sales level, promising to adjust its advertising spending accordingly. In the second stage, firms choose the actual levels of their advertising budget or sales target. When prices are exogenous, we show that, due to strategic effects, if a firm benefits from its rival's advertising (as when advertising increases awareness of the product category) then setting an advertising budget dominates setting a sales target. On the other hand, if a firm is harmed by its rival's advertising (as when advertising increases the firm's share of a fixed market), then committing to a sales level dominates. We extend these results in several directions and show that when firms engage in price competition as well as advertising the nature of advertising and product-market competition interact to determine whether setting an advertising budget or sales target dominates.

Keywords Advertising · Pricing · Marketing strategy · Game theory

JEL classifications M37 · M31 · D43 · C72

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1 Introduction

Choosing advertising expenditures is a fundamental marketing decision. Given the enormous stakes involved (Bass 1979) one would expect firms to employ state-of-the-art modeling practices to aid their decisions. However, many executives view the advertising budgeting process as highly complex, poorly structured, and very risky. Consequently, they use rigorous marketing models only as a supplement to simple rules of thumb (Farris et al. 1998).

The two most common methods of determining advertising expenditures are the percentage of sales and objective and task approaches. Under the percentage of sales approach, firms (such as retailers and service providers) commit at the beginning of the fiscal year to a budget that is a specified percentage of predicted (or the previous year's) dollar sales.[1] On the other hand, firms that follow the objective and task approach avoid precommitting to a budget in favor of making a detailed specification of sequential, measurable goals such as reach, frequency, production costs and even desired sales levels for the campaign. The popularity of this method, which has been adopted by firms such as Anhueser Busch and Unilever, is rapidly increasing.[2]

Marketing researchers have extensively investigated the optimal allocation of an advertising budget in noncompetitive (i.e., monopoly) settings. Using the concept of a market response function to parsimoniously capture the relationship between advertising spending and unit sales, researchers have applied sophisticated optimization algorithms to the optimal-budget problem.[3] Although these models have provided valuable insights (e.g., Sasieni (1971), Simon (1982), Rao (1986), Mahajan and Muller (1986)), their applicability is limited by the fact that most firms operate in competitive settings. In order to address this shortcoming, researchers have devised market response functions and models, which allow them to calculate and estimate equilibrium advertising spending by competing firms in environments in which firms do not take their rivals' advertising expenditures as fixed.[4] However, the market response function approach, like the majority of the literature, does not consider cases in which firms compete by setting both advertising and prices. In fact, very few papers consider both pricing and advertising. Notable exceptions include Villas-Boas (1993), which solves for the Markov perfect equilibria of advertising and pricing in a repeated duopoly interaction, and Iyer et al. (2005), which investigates the effect of targeted advertising on pricing and profits in a competitive environment.

In this article, we introduce a new strategic consideration into the competitive advertising problem. Specifically, we focus on the question of whether a firm should declare an advertising budget (as in the percentage of sales method) or a goal such as a sales target (as in the objective and task approach). We present the implications of

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[1] Fairhurst et al. (1996) found that 84% of service providers use this method.

[2] Mitchell (1993) found that about 40% of the UK manufacturing firms participating in his study used this method versus only 27% using the percentage of sales technique.

[3] See for example Little (1979) and Hanssens et al. (2001) for a review of specifications and estimation techniques.

[4] See Erickson (1985), Park and Hahn (1991), Chintagunta and Jain (1995), Bass et al. (2005), Dube et al. (2005).

this decision for the designing firm as well as its rival, and we characterize the conditions under which one approach or the other dominates.

The question of whether to set an advertising budget or sales target arises frequently in the real world. For example, when launching a new baking soda and peroxide toothpaste in 1994, Proctor & Gamble declared its objective to be "recognized in every household"; while Colgate-Palmolive declared that it would spend $40 million advertising its product.[5] On the other hand, when introducing competing products in 1997, P&G declared a $65 M campaign for Crest MultiCare and Colgate immediately followed by announcing a $100 M advertising budget for its Total.[6] Strategically, was it better for P&G to commit to a sales goal or an advertising budget?

To take a different example, when facing potential competition from IBM's OS2 Warp, Microsoft committed $100 million to the pre-launch advertising of Windows 95 without declaring a specific sales target. What effect might this declaration have had on IBM? Did the announcement make IBM's response more or less aggressive than it would have been if Microsoft had instead declared a commitment to selling 50 or 100 million units?

As these examples show, firms often make choices between advertising budgets and output/outcome targets. We argue that such choices have potentially important strategic effects. In light of this, we seek to characterize these strategic effects and provide a theory of when it is optimal for a firm to adopt one or the other posture.

In our basic model, the two firms compete in a two-stage game in which, in the first stage, each firm decides whether to compete by choosing an advertising budget (as in the percentage of sales approach) or by choosing a target sales level (as in the objective and task approach). While in the first stage firms decide which approach to adopt, they do not yet choose the specific value of their sales level or advertising budget.[7] In the second stage, the firms compete in a simultaneous-move game in which each firm, knowing which approach its rival has chosen, selects the specific value of its chosen strategic variable. Our primary concern is whether it is better to choose to set an advertising budget or a sales level in the first stage. Consequently, we focus on what we call the "advertising-sales game," the 2×2 game in which the players' strategies are whether to be an "advertising setter" or "sales setter," and the payoffs are the Nash equilibrium payoffs in the resulting second-stage game.[8]

The main result of this paper shows that each firm has a dominant strategy in the advertising-sales game. Whether setting an advertising budget dominates setting a sales level or vice versa depends on the nature of the competition between the firms. For expositional ease, call the firms Firm 1 and Firm 2 (the roles are completely reversible). If advertising by Firm 2 increases Firm 1's sales (the increase in the total

[5] Colgate packs $40 M behind new toothpaste; Advertising Age, Chicago; Dec 12, 1994.

[6] Business First, December 18, 1998.

[7] Throughout the paper we distinguish between choosing a variable (e.g., advertising budgets or sales levels) versus choosing the specific value of the variable (e.g., spend $2 million on advertising or achieve $100 million in sales).

[8] That is, if one player chooses to set an advertising budget and the other chooses to set a sales level, we compute the resulting equilibrium of the second-stage game when the competition is represented in terms of these variables. Thus, equilibrium of the advertising-sales game corresponds to a subgame perfect equilibrium of the two-stage game.

market by both firms' advertising supersedes the market share effects), then it is a dominant strategy for Firm 1 to choose to set an advertising budget. On the other hand, when advertising by Firm 2 decreases Firm 1's sales (market share competition), it is a dominant strategy for Firm 1 to choose to set a sales target. Due to these dominance relations, the advertising-sales game always has a unique pure strategy equilibrium in which the players adopt their dominant strategies.

The key to the results is that by adopting a sales-level strategy, a firm induces its opponent to act less aggressively (i.e., advertise less) than it would if the firm had adopted an advertising-budget strategy. This is because when a firm commits to a sales-target strategy, it promises to adjust its advertising in response to its rival in order to keep sales constant, and this adjustment tends to reduce the effectiveness of any increase in the rival's advertising expenditure (regardless of whether rival-advertising increases or decreases the firm's profit). When the rival's advertising increases the firm's profit, it wants the rival to spend a lot on advertising, and adopting an advertising-budget setting strategy encourages it to do so. On the other hand, when the rival's advertising decreases the firm's profit, it wants the rival firm to spend little on advertising. In this case, adopting a sales-target setting strategy accomplishes this goal.

We present several natural extensions to the basic model. First, we show that the main results continue to hold when there are n firms rather than two. Next, we consider a multi-period model of competition in order to incorporate the possibility that the advertising has lasting effects on the firms' profits and show that the main results persist in this environment. We then go on to consider the a model in which the effect of Firm 1's advertising on Firm 2 is qualitatively different than the impact of Firm 2's advertising on Firm 1. For example we show that when Firm 1's advertising increases Firm 2's sales while Firm 2's advertising decreases Firm 1's sales, setting an advertising budget is a dominant strategy for Firm 1 and committing to a sales level is dominant for Firm 2. A fourth extension addresses the fact that in many business examples a firm commits to its advertising budget only after observing the actions of its rival. We extend the model to allow for sequential declarations of the chosen variable and its level and show that the subgame perfect equilibrium has the firm that moves first committing to an advertising budget when advertising increases the total industry sales and committing to a sales level when advertising is used for market share gains. The final section of the paper considers a model in which firms compete by choosing both advertising and prices. Whether it is better to compete by setting an advertising budget or sales level in this case depends on the interaction between the intensity of price competition (i.e., how closely substitutable the goods are) and the nature of advertising competition (i.e., whether advertising helps or harms the rival firm and how strong these effects are).

This paper builds on Miller and Pazgal (2006), which considers general two-player games in which players may either compete by choosing input strategies or output strategies. Miller and Pazgal (2006) use general arguments to attack a general class of games. Consequently, although we often employ different proof techniques, some of the results mentioned in Section 2 follow as special cases of the results in that paper. However, many of the specific features that make whether firms should choose advertising budgets or sales targets an interesting and important marketing question are addressed here for the first time. In this paper, we specifically deal with these features, and make connections between our analyses and commonly employed marketing/advertising strategies, such as the percentage of sales and objective and task approaches.

The approach in this paper differs from typical models of advertising competition in that we consider not only firms' choices of strategies, but also their choices of what types of strategies they should choose. In effect, they are not only playing a game, but also choosing which game to play. Although this approach is novel in the study of advertising, similar questions have been considered in the strand of the industrial organization literature that considers whether firms should compete by setting prices (as in Bertrand competition) or quantities (as in Cournot competition). For example, Singh and Vives (1984) and Cheng (1985) consider the price vs. quantities question and show that for a range of demand specifications it is a dominant strategy for the players to choose to set quantities when the goods are substitutes and prices when the goods are complements.[9] The results in the price vs. quantity literature arise from the basic fact that, against a fixed price or quantity strategy by its opponent, a firm is indifferent between choosing a price or choosing a quantity. Thus, the importance of choosing to set a price or quantity lies in the strategic effects of adopting one or the other posture. Since firms react more aggressively (i.e., choose lower prices and higher quantities) when their rival sets price than when their rival sets quantity, this implies that when firms produce substitute products and want their rivals to be accommodating, they should set quantities. On the other hand, when they produce complements and want their rivals to be aggressive, they should set prices. Although the advertising vs. sales budget question addressed in this paper is qualitatively similar to the price vs. quantity question, the former question has important implications for actual marketing decisions and has not been previously considered.

The analysis presented in this paper is also related to the line of research initiated by Fudenberg and Tirole (1984) and Bulow et al. (1985), which consider the question of when firms will choose to engage in ex ante investments such as advertising or R&D that may affect the aggressiveness of rival firms. The results hinge on whether the firms compete in strategic substitutes or strategic complements (Bulow et al. 1985, p. 488), which is related to the question of whether increasing the ex ante investment makes product-market competition more or less aggressive. Our results are similar in spirit: whether it is better to set an advertising budget or a sales level depends on which posture induces more aggressive behavior in the product market and whether or not more aggressive behavior is desirable. However, the choice we consider is not how much to invest, but rather the firm's overall approach to their marketing strategy (i.e., whether to set an advertising budget or a sales level).

The paper proceeds as follows. Section 2 describes the model, and proves the initial results. Section 3 extends to n firm oligopoly setting, multi-period competition, asymmetric firms, and sequential commitments. Section 4 analyzes the three stage

[9] Klemperer and Meyer (1986) consider the impact of uncertainty on whether price- or quantity-setting is superior. Miller and Pazgal (2001) show that the distinction between price and quantity competition disappears when firms' owners are able to enter into sufficiently rich incentive contracts with their managers.

model including price setting and proves the main result. And finally, Section 5 offers a discussion.

2 The basic model

Consider an industry comprised of two competing firms. Each firm's sales level depends on a strictly increasing and strictly concave advertising response function that relates market sales to advertising spending by both participants (see Simon and Arndt (1980)). Although not needed for our results we use the common assumption that the function is bounded to prevent even the possibility of infinite sales. For our initial analysis we further assume a constant market price, p, relating sales to revenues.[10] In Section 4 we replace this feature of the model with the more realistic assumption that firms choose prices through a differentiated products Bertrand game. However, beginning with the fixed-price model allows us to cleanly identify the key strategic effect of budget- vs. target-setting, an effect which persists in a more complicated form in the model with pricing.

Throughout the paper, denote a generic firm by i and the competing firm by j. Denote firm i's sales by s_i and its advertising expenditure by a_i. Let $F(a_i, a_j)$ denote the sales by Firm i when its advertising budget is a_i and its rival's budget is a_j, i.e., $s_i = F(a_i, a_j)$. For the sake of simplicity we utilize a common single variable response function for both competing firms. The two most natural ways to define the sales response function for a duopoly are:

$$F\left(a_i, a_j\right) = f\left(a_i + t_i a_j\right), \text{and } G\left(a_i, a_j\right) = f\left(a_i\right) + t_j f\left(a_j\right),$$

where $f(\cdot)$ is a strictly increasing, strictly concave function and t_i measures the effect of Firm j's advertising on Firm i's sales. When $t_i>0$ Firm j's advertising is category expanding and increases Firm i's sales. In this case we refer to Firm j's adverting as expansive. Expansive advertising primarily increases the total size of the market. If $t_i<0$, Firm j's advertising decreases Firm i's sales, and we refer to Firm j's advertising as rivalrous.[11] Rivalrous advertising is primarily a tool for increasing market share. We refer to product categories for which advertising by any firm increases the rival sales as new or developing categories. While mature categories are identified by the negative impact of any firm's advertising on its rival's sales. To ensure that the effect of the rival firm's advertising can never be larger than the effect of the firm's own advertising spending, we assume $|t_i| < 1$.

The critical feature we are interested in is whether a firm will prefer to compete by setting sales or by setting an advertising budget. In order to consider this question, the model must allow us to represent competition as involving either both firms setting

[10] Assume that the price is either regulated or that each firm employs a constant margin strategy (Park and Hahn (1991)).

[11] Whether advertising is expansive or rivalrous is related to whether advertising competition is competition in strategic substitutes or strategic complements. See Bulow et al. (1985). However, the same relationship between the sign of t_i and the nature of competition does not hold across all types of competition we consider. Consequently, we adopt the terms "expansive" and "rivalrous" here because of their transparency and their usefulness across all of the models we consider.

advertising budgets, both firms setting sales level, or one firm setting an advertising budget and the other setting a sales level. The two specifications of the sales response function we consider specify a relation between the four variables a_1, a_2, s_1, and s_2 such that fixing any two of them determines the other two. Thus while the model contains four variables of interest, there are really only two degrees freedom. Hence we can consider competition in which both firms set advertising budgets (and sales levels are determined by the market), both firms set sales levels (and advertising budgets are determined by the market) or one firm sets an advertising budget and the other sets a sales target, with the remaining sales level and advertising expenditure being determined by the market. Firms' choices of which type of strategy to adopt will be driven by strategic considerations, i.e., the effect that adopting one or the other posture has on product-market competition and the firms' final profits.[12]

We start the analysis in Section 2.1 by showing that a monopoly is indifferent between choosing an advertising budget and a sales target, while Section 2.2 proves in detail that this is no longer the case in a competitive environment.

2.1 The monopoly case

In a monopolistic environment, a firm will chose an advertising level that maximizes its profit. To make the connection to the duopoly case that follows, let $f(a)$ denote sales level as a function of the monopolist's advertising expenditure, where $f(\cdot)$ is a strictly increasing, strictly concave function. Let $\Pi^a = ps - a = pf(a) - a$ denote the monopolist's profit. Assuming that the monopolist maximizes profit by choosing advertising expenditure, the optimal advertising level is given by the solution to the first order condition:

$$\frac{\mathrm{d}\Pi^a}{\mathrm{d}a} = pf'(a^*) - 1 = 0. \quad (1)$$

The monopolist could also solve its problem by choosing sales, with the chosen sales level dictating the necessary advertising level. In this case, the monopolist chooses s to maximize $\Pi^s = ps - a = ps - f^{-1}(s)$, where $f^{-1}(s)$ is the inverse of the advertising response function. The optimal choice of sales level is given by the solution to:

$$\frac{\mathrm{d}\Pi^s}{\mathrm{d}a} = p - \frac{\mathrm{d}}{\mathrm{d}s} f^{-1}(s^*) = 0. \quad (2)$$

The above condition can be rewritten as:

$$\frac{\mathrm{d}\Pi^s}{\mathrm{d}a} = p - \frac{1}{f'(f^{-1}(s^*))} = p - \frac{1}{f'(a^*)} = 0,$$

which is identical to Eq. 1. Hence, for a monopolist, the optimal outcome is the same regardless of whether it sets an advertising budget or sales target (and lets the other variable be determined by the market).

In this paper we show that this equivalence does not hold in a competitive setting. When strategic considerations are important, a firm's choice of, say, an advertising

[12] Of course, it may be that the firm is more able to commit to one or the other type of strategy. We discuss this issue later.

budget not only ties it to a decision variable but also conveys information to its rival about how it will react to the rival's behavior. This strategic effect leads the outcomes under budget-setting and sales-target-setting to diverge.

2.2 The duopoly case

Our analysis of the duopoly case considers two-stage games of the following form. In the first stage, each firm chooses whether to set a sales (profit) target or an advertising budget. After the choices have been made, the two decisions become common knowledge. In the second stage, the firms compete by simultaneously setting the specific values of their chosen advertising or sales variables. We are interested both in the Nash equilibria of the four second-stage games and in the subgame perfect equilibrium of the two-stage game. In particular, we seek to determine the effect of strategic considerations on the firms' decisions whether to set advertising budgets or a sales targets.

Consider the case where the advertising response function is given by $s_i = F(a_i,a_j)$. For simplicity, we assume that the firms are symmetric, i.e., $t_1=t_2=t$. This assumption is relaxed later. We begin by analyzing the second stage of the game, characterizing the equilibrium in all possible subgames: both firms choose advertising budgets, both choose sales levels, and the mixed cases where one firm chooses an advertising budget and the other chooses a sales level. Throughout the paper, we assume that the firms' best responses are always interior, i.e., that optimal strategies are strictly positive and that the first-order conditions for an optimum hold with equality. Allowing for corner solutions complicates the results but does not qualitatively alter them.

2.2.1 Both firms set advertising budgets

We use the superscript "aa" to indicate profit, advertising levels, etc. in the case where both firms set advertising budgets. In this case, firm i's profit is given by:

$$\Pi_i^{\mathrm{aa}} = \mathrm{ps}_i - a_i = \mathrm{pF}_i\left(a_i, a_j\right) - a_i = \mathrm{pf}\left(a_i + \mathrm{ta}_j\right) - a_i.$$

The equilibrium advertising strategies solve the following first order conditions:[13]

$$\mathrm{pf}'(a_1 + \mathrm{ta}_2) = 1, \text{and } \mathrm{pf}'(a_2 + \mathrm{ta}_1) = 1. \tag{3}$$

Since $f(x)$ is strictly concave and increasing, $f'(x)$ is invertible. Let $g(y)$ be this inverse of $f'(x)$: $g(f'(x))=x$. Using $g(y)$, the equilibrium advertising levels can be written as:

$$a_F^{\mathrm{aa}} \equiv a_{1F}^{\mathrm{aa}} = a_{2F}^{\mathrm{aa}} = \frac{g\left(\frac{1}{p}\right)}{1+t}. \tag{4}$$

[13] Throughout this subsection the second order conditions for maxima hold due to the concavity of $f(x)$.

2.2.2 Both firms set sales-levels

We use the superscript "ss" to indicate profit, advertising levels, etc. in the case where both firms set sales levels. When both firms commit to set sales targets, s_1 and s_2, we need to invert the sales advertising relationship in order to get advertising budgets as a function of sales levels. Inverting

$$s_1 = F(a_1, a_2) = f(a_1 + \mathrm{ta}_2), \text{and } s_2 = F(a_2, a_1) = f(a_2 + \mathrm{ta}_1),$$

yields:

$$a_1 = \frac{f^{-1}(s_1) - \mathrm{tf}^{-1}(s_2)}{1 - t^2}, \text{and } a_2 = \frac{f^{-1}(s_2) - \mathrm{tf}^{-1}(s_1)}{1 - t^2}.$$

Thus, firm i's profit is given by:

$$\Pi_i^{ss} = \mathrm{ps}_i - \frac{f^{-1}(s_i) - \mathrm{tf}^{-1}(s_j)}{1 - t^2}.$$

The equilibrium sales level strategies satisfy the following first order conditions:

$$p = \frac{1}{1 - t^2} \frac{\mathrm{d}}{\mathrm{d}s_i} \left(f^{-1}(s_i) \right). \tag{5}$$

Since $\frac{\mathrm{d}}{\mathrm{d}s}(f^{-1}(s)) = \frac{1}{f'(f^{-1}(s))}$, condition 5 can be rewritten as:

$$\mathrm{pf}'\left(f^{-1}(s_i)\right) = \mathrm{pf}'\left(a_i + \mathrm{ta}_j\right) = \frac{1}{1 - t^2}.$$

Combining the first-order conditions for both firms yields the equilibrium advertising levels:

$$a_F^{\mathrm{ss}} \equiv a_{1F}^{\mathrm{ss}} = a_{2F}^{\mathrm{ss}} = \frac{g\left(\frac{1}{1-t^2} \frac{1}{p}\right)}{1 + t}. \tag{6}$$

Note that, although we characterize the equilibrium advertising expenditure in this game, the strategic variables are indeed sales targets. The equilibrium strategies (sales targets) are given by $s_F^{\mathrm{ss}} = f\left(g\left(\frac{1}{1-t^2} \frac{1}{p}\right)\right)$ for each firm.

Comparing Eqs. 4 and 6 gives rise to the following proposition:

Lemma 1 *The equilibrium advertising levels when both firms compete by choosing advertising budgets is higher than the equilibrium advertising level when both firms compete by setting sales targets* $\left(a_F^{\mathrm{aa}} > a_F^{\mathrm{ss}}\right)$.

Proof All proofs are relegated to the Appendix. ■

To illustrate the intuition behind the proposition, consider Firm 1. Firm 1 cares about Firm 2's advertising spending only because of its effect of Firm 1's sales, and Firm 1 does not (directly) care about Firm 2's sales level at all. If Firm 1 knows that Firm 2 has set a sales level, then it believes Firm 2 will react to any change in Firm 1's advertising by changing its own advertising in order to maintain its desired sales level. For example, if $t>0$ then increasing a_1 increases s_2. If Firm 2 has set a sales level, then Firm 1 assumes that Firm 2 will respond to the increase in a_1 by decreasing a_2, which will in turn decrease s_1. On the other hand, if Firm 2 had set an advertising budget, Firm 2 would not react to Firm 1's increased advertising. Thus, Firm 1 perceives that increasing its advertising will be less effective when Firm 2 sets a sales level than when Firm 2 sets an advertising budget. Since the marginal benefit of increasing advertising is lower when Firm 2 commits to a sales level, Firm 1 is less inclined to increase its advertising, and its advertising spending is therefore smaller than when Firm 2 sets an advertising budget. Since both firms share this incentive, advertising when both set a sales level is smaller than when both set an advertising budget.

2.2.3 Mixed competition

Finally, we deal with the asymmetric case in which Firm 1 sets an advertising budget and Firm 2 sets a sales level. We use the superscript *as* to denote quantities in this case.

The equilibrium advertising levels are found by inverting the market response functions and expressing the two decision variables, s_1 and a_2, in terms of a_1 and s_2. Solving for the equilibrium levels of a_{1F}^{as} and a_{2F}^{as}. We get:[14]

$$a_{1F}^{\text{as}} = \frac{a_F^{\text{ss}} - \text{t}a_F^{\text{aa}}}{1-t}, \text{and } a_{2F}^{\text{as}} = \frac{a_F^{\text{aa}} - \text{t}a_F^{\text{ss}}}{1-t}. \quad (7)$$

Note that we express the asymmetric equilibrium advertising levels in terms of the equilibrium advertising levels in the two symmetric cases in order to ease our later comparisons.[15]

2.2.4 Results

We are now ready to state our first results comparing competition by setting advertising budgets with competition by setting sales targets:

Lemma 2 *Firm i*'s *equilibrium advertising budget is larger in the subgame where Firm j sets an advertising budget than in the subgame where Firm j sets a sales level. (Regardless of whether Firm i sets an advertising budget or a sales level.)*[16]

The proof for Lemma 2 is by straightforward computation. The intuition is similar to the intuition behind Lemma 1. All else being equal, Firm *i* will choose to advertise less when its opponent sets a sales level than when it sets an advertising budget. Starting from the case where Firm *j* sets advertising, moving to the case where Firm *j* sets a sales level gives Firm *i* an incentive to choose a smaller advertising

[14] Complete details are available upon request.

[15] As in the sales-setting subgame, note that although we characterize the equilibrium advertising expenditures here, Firm 2's strategic variable is a sales target, not an advertising budget. Firm 2's equilibrium strategy is to choose $s_{2F}^{\text{as}} = f\left(a_{2F}^{\text{as}} + ta_{1F}^{\text{as}}\right) = f\left((1+t)a_F^{\text{aa}}\right)$.

[16] The result stated here is about equilibrium outcomes but as we show in Section 4 we can prove a stronger result regarding the entire reaction functions.

expenditure (since, in adjusting its own advertising to maintain its sales level, a sales-level setting opponent will reduce the benefit of the firm's advertising expenditure). Of course, this change in behavior on the part of Firm i induces Firm j to change its behavior, which, in turn, induces a secondary effect on Firm i's advertising choice. However, whether these secondary effects are positive or negative, they are not sufficient to overwhelm the primary strategic effect: a firm advertises less when it faces a sales-target setting opponent than when it faces an advertising-budget setting opponent.[17]

We now turn to characterizing the equilibrium of the sales-advertising game and stating the main results of our paper. In order to facilitate the analysis we explicitly write the payoff matrix for the sales-advertising game.

Careful observation of the payoff matrix below (Eq. 8) reveals the following dominance relations:

Proposition 1 *In markets where advertising is expansive ($t>0$), setting an advertising budget is a dominant strategy (for each firm) in the sales-advertising game. On the other hand, when advertising is rivalrous ($t<0$), committing to a sales level is a dominant strategy.*

Firm 2

		Sets a_2	Sets s_2
Firm 1	Sets a_1	$\Pi_1^{aa} = pf((1+t)a^{aa}) - a^{aa}$ $\Pi_2^{aa} = pf((1+t)a^{aa}) - a^{aa}$	$\Pi_1^{as} = pf((1+t)a^{ss}) - \frac{a^{ss}-ta^{aa}}{1-t}$ $\Pi_2^{as} = pf((1+t)a^{aa}) - \frac{a^{aa}-ta^{ss}}{1-t}$
	Sets s_1	$\Pi_1^{sa} = pf((1+t)a^{aa}) - \frac{a^{aa}-ta^{ss}}{1-t}$ $\Pi_2^{sa} = pf((1+t)a^{ss}) - \frac{a^{ss}-ta^{aa}}{1-t}$	$\Pi_1^{ss} = pf((1+t)a^{ss}) - a^{ss}$ $\Pi_2^{ss} = pf((1+t)a^{ss}) - a^{ss}$

(8)

Proposition 1 implies that the sales-advertising game has a dominant strategy equilibrium. As described in the introduction, the intuition for the above proposition relies on the fact that expansive ($t>0$) advertising leads to a potential free-riding problem, where each firm might be tempted to limit its spending on advertising and rely on the other's. The way to alleviate the problem is by having each firm commit to an advertising budget, thus guaranteeing the rival that they do not intend to free ride. Conversely, rivalrous advertising ($t<0$) could lead to a destructive cycle. As a firm increases its advertising it harms its rival. In response, the rival firm will increase its own advertising, which may induce the firm to increase its advertising even further. In the end, the firms may find themselves with similar sales but substantially higher advertising expenditures than before the "advertising war" began. The way to mitigate this effect is by a commitment to a sales level, where each firm is (credibly) declaring that it will meet any increase in the other's advertising with a corresponding increase of its own necessary to maintain the specified sales level. Since this

[17] This phenomenon is examined in greater detail in Section 4, where, due to the added complexity of the model, the equilibrium advertising levels cannot be compared via direct computation.

compensation is harmful to the other firm, it will be less willing to increase its own advertising to begin with.

An interesting question is whether choosing the dominant strategy actually hurts the firms by lowering their profit versus the alternative of both choosing the dominated strategy. Proposition 2 summarizes the results.

Proposition 2 *In the sales-advertising game:*

A. When advertising is rivalrous ($t<0$) both firms committing to sales levels offers both firms the highest possible payoffs.
B. When advertising is expansive ($t>0$) then both firms choosing advertising levels yield higher profits than when both firm commit to sales targets but the equilibrium does not yield the highest potential profits.

When the advertising response function is given by its alternative formulation: $G_i(a_i, a_j) = f(a_i) + tf(a_j)$, it is straightforward to verify that Lemmas 1, 2 and Propositions 1, 2 hold. [18]

3 Extensions

In Section 2 we presented the strategic trade-off between setting an advertising budget and a sales target in a simple simultaneous duopoly environment. This section extends the previous results to more general and realistic environments. We start in Section 3.1 by showing that our results hold for more than two firms. In Section 3.2 we investigate the impact of allowing a multi-period interaction between the firms with lingering effects of advertising. In section 3.3 we relax the symmetry assumption between the firms and allow for different impact of each firm's advertising on its rival sales. Finally, Section 3.4 shows that the previous results hold even in the common case where one firm chooses a particular value of an advertising budget or sales target and the rival firm then responds optimally.[19]

3.1 Oligopoly with n firms

We now present a generalization of the previous results to an oligopolistic environment with n symmetric firms where the sales response functions are constructed as:

$$s_i = F(a_1, \cdots, a_n) = f\left(a_i + \frac{t}{n-1}\sum_{j\neq i} a_j\right) \text{for } i = 1, \cdots, n.$$

Using techniques similar to the ones presented in Subsections 2.2.1 and 2.2.2, we derive the equilibrium advertising levels when all n firms choose an advertising budget, $a_F^{aa} = \frac{g\left(\frac{1}{p}\right)}{1+t}$, as well as when all n firms commit to a sales target, $a_F^{ss} = \frac{g\left(\frac{(n-1+(n-2)t)}{(n-1-t)(1+t)}\frac{1}{p}\right)}{1+t}$. From which the generalization of Lemma 1 follows, however, in order to prove Lemma 2 and

[18] Explicit computations available upon request.

[19] Throughout this section we provide extensive proofs only for the advertising response function being $F(a_i,a_j)$ but all the results can be easily shown to be true for $G(a_i,a_j)$ as well.

Propositions 3–4 we need to calculate the equilibrium advertising levels when k firms choose advertising budgets and the other $(n-k)$ choose a sales target. These are given by $a_F^{a_k s_{n-k}} = \frac{t(k-n)(g_2-g_1)+(n-1-t)g_1}{(n-1-t)(t+1)}$ for the firms choosing advertising budgets and $a_F^{s_{n-k} a_k} = \frac{tk(g_2-g_1)+(n-1-t)g_2}{(n-1-t)(t+1)}$ for the firms committing to a sales target (where $g_1 = g\left(\frac{(n-1)}{(n-1-t)}\frac{(n-k-1)t+n-1}{((n-k)t+n-1)}\right)$ and $g_2 = g\left(\frac{(n-1)}{(n-1-t)}\frac{((n-k-2)t+n-1)}{((n-k-1)t+n-1)}\right)$).[20] It is straightforward to check that given the above equilibrium advertising levels all of the results presented in Section 2 still hold. Specifically, we generalize Proposition 1:

Proposition 3 *When n firms are competing in markets where advertising is rivalrous ($t<0$), committing to a sales level is a dominant strategy (for all firms) in the sales-advertising game. On the other hand, when advertising is expansive ($t>0$), setting an advertising budget is a dominant strategy.*

3.2 Multi-period commitment

In many competitive environments the commitment to an advertising budget or a sales target is declared at the beginning of the year while decisions as to the specific levels of these parameters are made gradually over the season. Furthermore, a substantive body of research has shown that advertising has a lingering but declining effect over time (for an extensive survey see Stewart and Kamins (2002)). Specifically, advertising by a firm in one period has an impact on its (and its rival's) sales not only at the time of advertising but in the future as well. We capture this phenomenon with a two-period selling season. Prior to the first period, each firm chooses whether to commit to an advertising budget or a sales level. After this commitment is revealed, each firm chooses the first period level of its variable and subsequently the second-period level taking into account the fact that a fraction $\lambda \in (1,0)$ of its first period advertising will spill over and increase the impact of its second period advertising level. We assume that after each stage's choices have been made, the decisions become common knowledge. We are interested in characterizing the subgame perfect equilibrium of the described game.

In order to investigate the commitment choice in the preliminary period we need to consider all four potential subgames (both firms commit to an advertising budget, both commit to a sales target, and one firm commits to an advertising budget while the other commits to a sales target). For each subgame we need to sequentially calculate the equilibrium levels of the chosen strategic variables for both decision stages.

When both firms set an advertising budget in the preliminary stage, the second period profit maximized by each one is:

$$\prod\nolimits_{i,2}^{\mathrm{aa}} = \mathrm{ps}_{i,2} - a_{i,2} = \mathrm{pf}\left(a_{i,2} + \lambda a_{i,1} + t\left(a_{j,2} + \lambda a_{j,1}\right)\right) - a_{i,2},$$

where $a_{i,k}$ represents firm i's advertising spending in stage k. This leads to the following equilibrium advertising budgets in the second stage: (as a function of first

[20] Explicit calculations are available upon request.

stage values) $a_{i,2}^{aa} = \frac{g\left(\frac{1}{p}\right)}{1+t} - \lambda a_{i1}^{aa}$. Substituting the result into the total two-period profit function we get for each firm:

$$\prod_i^{aa} = \text{pf}\left(a_{i1} + \text{ta}_{j1}\right) - a_{i1} + \text{pf}\left(g\left(\frac{1}{p}\right)\right) - \frac{g\left(\frac{1}{p}\right)}{1+t} + \lambda a_{i1}.$$

The equilibrium first period advertising budgets are identical for both firms and are given by $a_{i1}^{aa}(\lambda) = \frac{g\left(\frac{1-\lambda}{p}\right)}{1+t}$ and the second period advertising equilibria are given by $a_{i2}^{aa}(\lambda) = \frac{g\left(\frac{1}{p}\right) - \lambda g\left(\frac{1-\lambda}{p}\right)}{1+t}$.

Similarly, the optimal levels of advertising budgets when both firms commit to sales target are given by:

$$a_{i,1}^{ss}(\lambda) = \frac{1}{1+t} g\left(\frac{1}{1-t^2}\frac{1-\lambda}{p}\right) \text{and } a_{i,2}^{ss}(\lambda) = \frac{1}{1+t} g\left(\frac{1}{1-t^2}\frac{1}{p}\right) - \lambda a_{i,1}^{ss}(\lambda).$$

Finally when firm i commits to an advertising budget while firm j commits to a sales target the optimal advertising budgets are

$$a_{i,1}^{as}(\lambda) = \frac{g\left(\frac{1}{1-t^2}\frac{1-\lambda}{p}\right) - \text{tg}\left(\frac{1-\lambda}{p}\right)}{1-t^2} \text{and } a_{i,2}^{as}(\lambda) = \frac{g\left(\frac{1}{1-t^2}\frac{1}{p}\right) - \text{tg}\left(\frac{1}{p}\right)}{1-t} - \lambda a_{i,1}^{as}(\lambda), \text{and}$$

$$a_{j,1}^{as}\lambda) = \frac{g\left(\frac{1-\lambda}{p}\right) - \text{tg}\left(\frac{1}{1-t^2}\frac{1-\lambda}{p}\right)}{1-t^2} \text{and } a_{j,1}^{as}(\lambda;) = \frac{g\left(\frac{1}{p}\right) - \text{tg}\left(\frac{1}{1-t^2}\frac{1}{p}\right)}{1-t} - \lambda a_{j,1}^{as}(\lambda).$$

Inspection of the optimal advertising levels shows that if advertising does not have a lingering effect (as λ=0) the optimal advertising levels are identical for both periods and coincide with the levels found in Subsection 2.2. For every λ>0 the first period advertising is larger than its single-period counterpart due to the fact that advertising increase sales not only in the first period but in the second period as well. On the other hand, second period advertising levels are smaller than their one period counterparts. Furthermore, all of the results presented in section 2.2.4 still hold. Specifically:

Proposition 4 *When advertising is expansive (t>0), setting an advertising budget is a dominant strategy (for each firm and every period) in the multi-period sales-advertising game. On the other hand, when advertising is rivalrous (t< 0), committing to a sales level is a dominant strategy (again for each firm and every period).*

The intuition behind the Proposition 4 follows from the fact that in the last stage of the game the firms face a competitive problem that is identical to the one analyzed in Subsection 2.2. Consider, for example, the case where t>0. We know from Theorem 3 that in this case choosing an advertising budget is a dominant strategy for each firm.[21] Clearly when thinking about the first period choice of advertising budget or sales target we know that if t>0 choosing advertising budget is best for that period. Furthermore, Lemma 1 guarantees that commitment to an advertising

[21] Technically, the firms face a sales response function of the form $F_i(a_{i,2}, a_{j,2}) = f(h_i + a_{i,2} + ta_{j,2})$ where $h_i = \lambda(a_{i,1} + ta_{j,1})$ is a constant. Proposition 1 is only proved for response functions of the form $f(h + a_{i,2} + ta_{j,2})$ but can be easily extended to handle firm specific constants.

budgets leads to higher level of advertising spending which also increase the carry over effects for the second period.

The results of this subsection clearly hold when we introduce discounting of future revenue streams and may be generalized to any finite number of periods.

3.3 Asymmetric advertising effects

The results of the previous section also extend to the case where the firms' advertising effects are not necessarily symmetric. The most tractable way to achieve this is to allow for $t_1 \neq t_2$. Specifically, let:

$$s_1 = f(a_1 + t_1 a_2), \text{and } s_2 = f(a_2 + t_2 a_1).$$

Using the same methods as in the previous section, the optimal advertising budgets under the different types of competition are given by:[22]

$$a_i^{\text{aa}} = g\left(\frac{1}{p}\right)\frac{1-t_i}{1-t_i t_j},\ a_i^{\text{ss}} = g\left(\frac{1}{1+t_i t_j}\frac{1}{p}\right)\frac{1-t_i}{1-t_i t_j},$$
$$a_1^{\text{as}} = \frac{g\left(\frac{1}{1-t_1 t_2}\frac{1}{p}\right) - t_1 g\left(\frac{1}{p}\right)}{1-t_1 t_2}, \text{and } a_2^{\text{as}} = \frac{g\left(\frac{1}{p}\right) - t_2 g\left(\frac{1}{1-t_1 t_2}\frac{1}{p}\right)}{1-t_1 t_2}. \quad (9)$$

Inspection of Eq. 9 reveals that if the effect of one firm's advertising on its rival's sales has the same sign for both firms (i.e., $t_1 t_2 > 0$), then all the results of the previous section hold. On the other hand we now face the new possibility that advertising by one firm increases sales of its rival, but advertising by the other firm decreases the sales of the original one (i.e., $t_1 t_2 < 0$).

As an example, consider an advertising campaign by Tiffany's for an expensive diamond ring. A successful campaign will raise the awareness of people as to the importance of giving jewelry as a present, but some consumers' budget constraints may lead them to search for a more economic alternative, for example at a local Kay jeweler. On the other hand, it is very unlikely that customers exposed to advertising by Kay will seek to purchase at Tiffany's. In fact, Kay's advertising message might convince some potential Tiffany's customers to opt for the cheaper alternative. Thus Tiffany's advertising is expansive while Kay's is rivalrous. A different example might include a competition between a national chain and a local store. Advertising by a popular video rental chain such as Blockbuster promoting a specific video may increase business at all video rental stores, while advertising by one of the small local stores will primarily steal business away from the chain.

The main results change slightly when $t_1 t_2 < 0$. In contrast to Lemma 1, the equilibrium advertising budgets are smaller when the rival firm commits to an advertising budget than when it commits to a sales level.

Proposition 5 *If $t_1 t_2 < 0$ then, Firm i's optimal advertising budget is smaller when Firm j sets an advertising budget then when it commits to a sales target.*

[22] Note that Eq. 9 reduces to Eqs. 4, 6, and 7 when $t_1 = t_2$.

Moreover, in contrast to Proposition 1, we show in Proposition 6 below that in equilibrium it is a dominant strategy for the firm whose advertising decreases its rival's sales to commit to sales target while it is a dominant strategy for the other to choose an advertising budget.

Proposition 6 *If Firm 1's advertising increases Firm 2's sales ($t_2 > 0$) while Firm 2's advertising decreases Firm 1's sales ($t_2 < 0$), then setting an advertising budget is a dominant strategy for Firm 1 and committing to a sales level is dominant for Firm 2.*

Returning to our previous jewelry example, the equilibrium will involve Tiffany's choosing an advertising budget and the local Kay jeweler choosing a sales level. By committing to a sales level, the local jeweler signals to Tiffany's that it will not attempt to steal too much of its business, and by committing to an advertising budget Tiffany's signals that it will not free ride off of the local store's generosity.

3.4 A sequential version of the game

In many real world cases, one firm announces either a specific advertising budget or sales level and then the other firm responds. There are two ways in which our basic model does not capture these interactions. First, the firms interacted sequentially, not simultaneously. Second, the firms did not first make announcements such as "I will set a sales target" or "I will set an advertising budget." They simply announced the values of their strategic variables.

In this section, we show that the qualitative results of our basic model extend to cases in which, prior to any action by Firm 2, Firm 1 has the opportunity to credibly commit not only to setting an advertising budget or a sales level but to a particular value (e.g., spend \$40 M on advertising or achieve \$200 M in sales). Following Firm 1's initial move, Firm 2 observes Firm 1's actions, and chooses a best response to Firm 1s action. We investigate the unique subgame perfect equilibrium (SPE) of this competitive environment (which can be found via backward induction).

We start by analyzing the case in which Firm 1 moves first and chooses an advertising budget. Suppose Firm 2 responds by setting advertising budget a_2 in order to maximize $\prod_2^{aa} = pf(a_2 + ta_1) - a_2$. Differentiating $\prod_2^{aa}$ with respect to a_2 yields optimality condition:

$$a_2 + ta_1 = g\left(\frac{1}{p}\right), \tag{10}$$

which is the same as in the simultaneous version of the game in which both parties set advertising budgets. Using backward induction, Firm 1's optimal choice of an advertising budget is determined by maximizing:

$$\prod_1^{aa} = pf(a_1 + t(g(1/p) - ta_1)) - a_1 = pf\left(\left(1 - t^2\right)a_1 + tg(1/p)\right) - a_1,$$

which yields optimal advertising budget (for Firm 1):

$$a_1^{aa} = \left(g\left(\frac{1}{1-t^2}\frac{1}{p}\right) - tg\left(\frac{1}{p}\right)\right) \Big/ \left(1 - t^2\right).$$

This implies Firm 2's equilibrium advertising budget is:

$$a_2^{aa} = g\left(\frac{1}{p}\right) - ta_1^{aa} = \frac{g\left(\frac{1}{p}\right) - tg\left(\frac{1}{1-t^2}\frac{1}{p}\right)}{(1-t^2)}.$$

If, instead, Firm 2 responds to Firm 1's advertising budget announcement by setting a sales level, it chooses s_2 in order to maximize:

$$\prod_2^{as} = ps_2 - \left(f^{-1}(s_2) - ta_1\right).$$

Firm 2's optimality condition in this case is given by $a_2^{as} + ta_1^{as} = g\left(\frac{1}{p}\right)$.

As expected, since this is exactly the same optimality condition as given by Eq. 10, Firm 2's optimal behavior does not depend on whether it chooses a budget or a sales target.

Regardless of Firm 2's choice of strategic variable, Firm 1's profit is given by:

$$\prod_1^{a\cdot} = pf\left(a_1^{a\cdot} + ta_2^{a\cdot}\right) - a_1^{a\cdot} = pf\left(g\left(\frac{1}{1-t^2}\frac{1}{p}\right)\right) - g\left(\frac{1}{1-t^2}\frac{1}{p}\right) - tg\left(\frac{1}{p}\right)\Big/(1-t^2).$$

Repeating the above analysis to the case when Firm 1 chooses to lead with a (particular) sales target, we get that Firm 1's profit in equilibrium is given by:

$$\prod_1^{s\cdot} = ps_1^{s\cdot} - a_1^{s\cdot} = pf\left(g\left(\frac{1}{1-t^2}\frac{1}{p}\right)\right) - \frac{g\left(\frac{1}{1-t^2}\frac{1}{p}\right)}{(1+t)}.$$

Given the above results, the following Proposition extends our basic analysis to the sequential case.

Proposition 7 *Consider the modification of the sales-advertising game where Firm* 1 *moves first and chooses a specific budget or sales target. The subgame perfect equilibrium is for Firm* 1 *to commit to an advertising budget when advertising is expansive ($t > 0$) and commit to a sales level when advertising is rivalrous ($t < 0$). Firm* 2 *is completely indifferent as to its choice of optimal advertising budget or sales level.*

Proposition 7 establishes a result similar to Proposition 1. A first-mover with the power to commit will choose an advertising budget when advertising is expansive and a sales target when advertising is rivalrous.

An example that includes both asymmetric effects and sequential interaction revolves around the competition between national and store brand at a local supermarket. The recent multimillion dollar television and magazine campaign by Proctor and Gamble to advertise Fabreze was mainly aimed to grow the size of fabric cleaners and probably benefited some generic store brands as well. Yet, in retaliation, local grocery stores started using internal advertising placed on the shelves stocking fabric softeners and pointing to the cheaper but equally potent store brand. According to the above theory P&G should indeed adopt an advertising budget (as it did) knowing that its advertising is likely to increase total category sales and help the competition as well. On the other hand, the local stores undoubtedly aim at rivalrous advertising trying to increase their market share at the expense of P&G. Our theory claims that a simultaneous campaign by the local chains would have optimally opted for a sales level target. However it is more likely that the stores' decided on advertising as a response to the announcement by P&G and thus the theory predicts indifference between a choice of a budget or a sales target. Indeed we have no information as to the declared goal of the local supermarket chains.

4 Differentiated-products price competition[23]

Until now, we have assumed that the firms take prices as given. Obviously firms need to determine not only their advertising level but also the prices they wish to charge for their products or services. In this section, we extend the basic model to this more realistic setting by allowing the firms to set prices as well as advertising budgets or sales levels. We show that the flavor of the basic results still holds. Specifically, the determinants of the optimality of commitment to an advertising budget or a sales level must include not only the impact of one firm's advertising on its rival sales but the direct affect of its prices as well.

Introducing price competition to our model, we now consider three-stage games. In the first stage, the firms choose whether to set advertising budget or a sales level. In the second stage, the firms set the value of the variable they chose in the first stage, and in the third and final stage they choose prices for their differentiated products and compete in the market.

In the product market, the firms produce differentiated products and compete by setting prices. For the sake of analytical tractability we present the results of this section for the advertising response function $G(a_1,a_2)$.[24] Demand for firm i's product is then given by:

$$q_i = G(a_i, a_j) - p_i + \mathrm{zp}_j \tag{11}$$

where z measures the substitutability of the products ($z>0$ implies substitute products while $z<0$ represents complementary products). The advertising response function determines the intercept of the demand function. Thus, increasing advertising increases own demand and increases or decreases demand for the other firm's product depending on whether t is positive or negative. As before, we are interested in the subgame-perfect equilibrium of this game. We solve for the equilibrium by using backward induction, starting with the solution to the pricing game given the advertising budgets, and then substituting this known solution into the original problem.

In our earlier analysis, the relationship between advertising expenditure and sales was straightforward. Because price was fixed, it was not important whether we thought of firms as committing to unit sales or sales revenue. In this version of the analysis, where pricing strategies also play a role, this distinction is important. For the sake of

[23] The arguments in this section parallel those of Miller and Pazgal (2006), extending that paper's analysis of two-stage games to include a third (pricing) stage.

[24] We adopt $G(a_i,a_j)$ in this section because we are interested in deriving results in terms of parameters t and z that allow us to investigate the interaction of advertising and product-market competition. While the qualitative results persist when the advertising response function is given by $F(a_i,a_j)$, the functional form proves difficult to invert analytically when both parameters are left free.

analytic tractability and realism, we now assume that a firm that adopts a sales-target strategy commits to a level of sales revenue rather than a unit-sales target.

4.1 The advertising-revenue relationship

We begin by deriving the relationship between advertising expenditure and sales revenue. Given advertising levels for both firms, the third-stage price competition implies that Firm i chooses p_i in order to maximize $\prod_i = p_i q_i - a_i$, which yields following price-reaction function:

$$p_i = \frac{1}{2}\left(G(a_i, a_j) + \mathrm{zp}_j\right) \tag{12}$$

Simultaneously solving for the two reaction function yields third-stage equilibrium prices (as functions of the advertising levels):

$$p_i = \left(2G(a_i, a_j) + \mathrm{zG}(a_j, a_i)\right)/\left(4 - z^2\right),$$

which implies third-stage sales revenue of:

$$s_i = \left(\frac{2G(a_i, a_j) + \mathrm{zG}(a_j, a_i)}{4 - z^2}\right)^2 = \left(\frac{(2 + \mathrm{zt})f(a_i) + (2t + z)f(a_j)}{4 - z^2}\right)^2. \tag{13}$$

Since Eq. 13 gives sales as a function of advertising expenditure, it provides the relationship we need in order to study the first and second stages of the game. Note that s_i is strictly increasing in a_i, and increases or decreases in a_j depending on whether ($2t$+z) is positive or negative. Since when ($2t$+z)=0 the firm's sales does not depend on the other firm's advertising, we will assume ($2t$+z)≠0 in order to focus on the more interesting cases.

4.2 The second stage

We now turn to the second-stage analysis. When both firms choose to set advertising budgets in the first stage, Firm i chooses a_i to maximize:

$$\prod\nolimits_i^{\mathrm{aa}} = \left(\left((2 + \mathrm{zt})f(a_i) + (2t + z)f(a_j)\right)/\left(4 - z^2\right)\right)^2 - a_i. \tag{14}$$

Differentiating $\prod_i^{\mathrm{aa}}$ with respect to a_i yields Firm i's (implicitly defined) reaction curve:[25]

$$2\left((2 + \mathrm{zt})f(a_i) + (2t + z)f(a_j)\Big/\left(4 - z^2\right)^2\right)(2 + \mathrm{zt})f'(a_i) = 1. \tag{15}$$

Next, consider the case in which both firms commit to setting a sales (revenue) target. Inverting Eq. 13 yields advertising as a function of sales:

$$a_i = f^{-1}\left(\left((2 + \mathrm{tz})\sqrt{s_i} - (2t + z)\sqrt{s_j}\right)/\left(1 - t^2\right)\right). \tag{16}$$

[25] In this and all versions of the problem we study, we assume that there is a unique solution to firm i's maximization problem. This will be the case whenever $f(a)$ is sufficiently concave.

Firm i's profit in this version of the game is given by:

$$\prod_i^{ss} = s_i - f^{-1}\left(\left((2+\mathrm{tz})\sqrt{s_i} - (2t+z)\sqrt{s_j}\right)/\left(1-t^2\right)\right).$$

Differentiating $\prod_i^{ss}$ with respect to s_i yields:

$$1 = \frac{(2+\mathrm{tz})}{2(1-t^2)\sqrt{s_i}} \frac{\mathrm{d}f^{-1}}{\mathrm{d}x}(x^*) = \frac{(2+\mathrm{tz})}{2(1-t^2)\sqrt{s_i}} \frac{1}{f'\left(f^{-1}\left(\frac{(2+\mathrm{tz})\sqrt{s_i}-(2t+z)\sqrt{s_j}}{1-t^2}\right)\right)}, \quad (17)$$

where $x^* = \frac{(2+\mathrm{tz})\sqrt{s_i}-(2t+z)\sqrt{s_j}}{1-t^2}$ and the second equality stems from the fact that $\frac{\mathrm{d}f^{-1}(x)}{\mathrm{d}x} = \frac{1}{f'(f^{-1}(x))}$. Substituting Eqs. 16 and 17 into Eq. 13 yields Firm i's (implicitly defined) reaction function:

$$2\left(\frac{(2+\mathrm{zt})f(a_i) + (2t+z)f(a_j)}{4-z^2}\right)\frac{(1-t^2)}{(2+\mathrm{tz})}f'(a_i) = 1. \quad (18)$$

As before, firm i's best response to firm j depends on whether j sets advertising or sales, but not on whether firm i does. To see this, consider the case where 1 sets a_1 and 2 sets s_2. Solving for s_1 and a_2 as functions of a_1 and s_2 allows us to write the profit functions in terms of these decision variables:

$$s_1 = \frac{\left((2t+z)\sqrt{s_2} + (1-t^2)f(a_1)\right)^2}{(2+\mathrm{zt})^2}, \text{and } a_2 = f^{-1}\left(\frac{(4-z^2)\sqrt{s_2} - (z+2t)f(a_1)}{2+\mathrm{zt}}\right). \quad (19)$$

When Firm 1 chooses an advertising budget taking firm 2's sales target as given, profit is given by:

$$\prod_1^{as} = \frac{\left((2t+z)\sqrt{s_2} + (1-t^2)f(a_1)\right)^2}{(2+\mathrm{zt})^2} - a_1$$

Differentiating with respect to a_1, Firm 1's first-order condition is:

$$\begin{aligned} 2(1-t^2)\frac{\left((2t+z)\sqrt{s_2} + \left(1-t^2\right)f\left(a_1^{as}\right)\right)}{(2+\mathrm{zt})}f'\left(a_1^{as}\right) &= (2+\mathrm{tz}) \\ 2(1-t^2)\sqrt{s_1}\, f'\left(a_1^{as}\right) &= (2+\mathrm{tz}) \\ 2\left(\frac{(2+\mathrm{zt})f(a_i) + (2t+z)f\left(a_j\right)}{4-z^2}\right)\frac{\left(1-t^2\right)}{(2+\mathrm{tz})}f'(a_i) &= 1 \end{aligned} \quad (20)$$

where the second line follows from the first by Eq. 19, and the third line follows from the second by Eq. 13. Condition 20 is equivalent to Firm 1's reaction function when both firms set advertising budgets (see Eq. 15). Thus the firm's optimal reaction does not depend on whether Firm 1 sets an advertising budget or a sales target.

Next, consider Firm 2 (which chooses a sales target). It maximizes:

$$\Pi_2^{\mathrm{as}} = s_2 - f^{-1}\left(\frac{(4-z^2)\sqrt{s_2} - (z+2t)f\left(a_1^{\mathrm{as}}\right)}{2+\mathrm{zt}}\right).$$

Differentiating Π_2^{as} with respect to s_2 yields:

$$\begin{aligned} 1 &= \frac{\mathrm{d}}{\mathrm{d}s_2}\left(f^{-1}\left(\frac{(4-z^2)\sqrt{s_2} - (z+2t)f\left(a_1^{\mathrm{as}}\right)}{2+\mathrm{zt}}\right)\right)\frac{(4-z^2)}{2+\mathrm{zt}}\frac{1}{2\sqrt{s_2}} \\ &= \frac{1}{f'\left(a_2^{\mathrm{as}}\right)}\frac{(4-z^2)}{2+\mathrm{zt}}\frac{1}{2\sqrt{s_2}} \end{aligned} \tag{21}$$

Using the definition of s_i, Eq. 21 can be rewritten as:

$$\begin{aligned} \frac{2\left(\sqrt{s_i}\right)(2+\mathrm{zt})f'(a_i)}{4-z^2} &= 1, \text{or} \\ 2\left(\frac{(2+\mathrm{zt})f(a_i)+(2t+z)f\left(a_j\right)}{(4-z^2)^2}\right)(2+\mathrm{zt})f'(a_i) &= 1, \end{aligned} \tag{22}$$

where the second line follows from Eq. 13. Since this expression is identical to Eq. 18, Firm 2's reaction to an advertising-setting opponent does not depend on whether it sets a_2 or s_2.

Given the firms' choices of strategic variables in the first stage, the equilibrium of the second and third stage of the game is determined by the intersection of the appropriate reaction curves (and the ensuing prices in the third stage). Hence if firm 1 sets advertising and firm 2 sets sales, the equilibrium advertising levels are found by solving Eqs. 20 and 22 for a_1 and a_2.[26]

One possible configuration of the equilibria is depicted in the following diagram, where R_i^y denotes Firm i's reaction curve to an opponent who sets $y \in \{a,s\}$. The equilibrium advertising levels are labeled x^{aa}, x^{as}, x^{sa}, and x^{ss}, where the first superscript denotes 1's strategic variable and the second superscript denotes 2's. Note that, x^{as}, the equilibrium advertising level when 1 sets a and 2 sets s is given by the intersection of R_1^s and R_2^a. This is a manifestation of the fact that 1's reaction curve depends on its opponent's strategic variable but not its own.

4.3 The first stage

In the first stage of the game, each firm chooses whether to be an advertising-budget-setter or sales-target-setter. Given a choice for each firm, the equilibrium advertising levels are given by x^{aa}, x^{as}, x^{sa}, and x^{ss}. Let $\pi_i(x)$ denote Firm i's sales revenue when advertising levels are x. Given equilibrium play in the second and third stages of the game, the remainder of the analysis focuses on the firms' incentives the first-stage game, which we call the sales-advertising game.

Suppose Firm 2 is an advertising-setter, and consider Firm 1's decision whether to set an advertising budget or a sales level. If 1 chooses to set advertising, the

[26] Although we solve for the equilibrium advertising expenditures, in the actual play of the game firm 2 sets a sales target. However, since specification of a_1 and s_2 uniquely determines a_2, there is no inconsistency or loss of generality in our approach.

equilibrium outcome is x^{aa}, while if it sets sales the equilibrium outcome is x^{sa}. Since in either case Firm 2 sets advertising, both of these points lay along R_1^a, Firm 1's reaction curve against an advertising-setting opponent. Thus, the critical determinant of whether Firm 1 prefers outcome x^{aa} or x^{as} is how Firm 1's profit changes along R_1^a. Similarly, if Firm 2 were an *s*-setter, the critical factor in determining whether Firm 1 prefers to set *s* (leading to outcome x^{ss}) or *a* (leading to outcome x^{as}) is how Firm 1's profit changes along R_1^s. The same considerations (with the roles reversed) determine Firm 2's preferences over x^{aa} vs. x^{as} and x^{sa} vs. x^{ss}.

In the Appendix, we prove three results that characterize how profit changes along the players' reaction curves R_i^a and R_i^s. They are:

Property 1: Firm *i*'s reaction to an *s*-setting opponent is smaller than its reaction an *a*-setting opponent.[27]
Property 2: The slope of the players' reaction curves is the same as the sign of $(2t+z)$.
Property 3: Along a player's reaction curve, profit increases in a_j if $(2t+z)>0$ and decreases in a_j if $(2t+z)<0$.

Property 1, which is a generalization of Lemma 2, says that Firm *i* chooses to spend more on advertising when its opponent is an advertising-setter than when its opponent is a sales-target-setter. This is because the marginal benefit to increasing advertising when the other firm sets advertising is greater than when the other firm sets a sales target. When Firm *i* increases a_i, this affects Firm *j*'s sales. If Firm *j* has committed to a sales target, it will have to vary a_j in order to maintain sales, and this change in a_j will tend to counteract the benefit to Firm *i* of the original increase in a_i. Intuitively, Properties 2 and 3 follow from the fact that increasing a_j increases the marginal profitability of an increase in a_i if and only if $(2t+z)>0$.

Properties 1–3 allow us to characterize the equilibrium of the sales-advertising game. Consider the case where $(2t+z)>0$. In this case, the players' various reaction curves are upward sloping and configured as in Fig. 1.[28] Therefore, Property 3 implies that profit increases as one moves upward along R_1^a or R_1^s, which in turn implies that $\pi_1(x^{as})>\pi_1(x^{ss})$ and $\pi_1(x^{aa})>\pi_1(x^{sa})$. That is, Firm 1 has a dominant strategy in the sales advertising game to choose to be an advertising-setter. Similarly, $\pi_2(x^{ss})>\pi_2(x^{sa})$ and $\pi_2(x^{as})>\pi_2(x^{aa})$, and thus Firm 2 also has a dominant strategy to be an advertising-setter.

If, on the other hand, $(2t+z)<0$, Property 2 implies that the firms' reaction curves are downward sloping, and so the diagram must resemble Fig. 2.

When $(2t+z)<0$, Property 3 implies that $\pi_1(x^{as})<\pi_1(x^{ss})$ and $\pi_1(x^{aa})<\pi_1(x^{sa})$, and that $\pi_2(x^{ss})<\pi_2(x^{sa})$ and $\pi_2(x^{as})<\pi_2(x^{aa})$. Hence, in this case, setting a sales target is a dominant strategy for each firm. Proposition 10 summarizes.

Proposition 10 *If $(2t+z)>0$, for either firm setting an advertising budget in the sales-advertising game dominates setting a sales target. If $(2t+z)<0$, then setting a sales target dominates setting an advertising budget.*

[27] That is, in Fig. 1, R_1^a lies to the right of R_1^s and R_2^a lies above R_2^s.

[28] They are drawn as linear for expositional purposes, but they need not be.

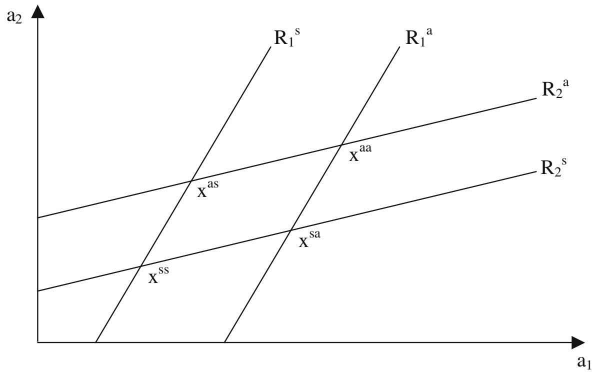


Fig. 1 The equilibria of the second-stage game

In the game with differentiated products and pricing, the results depend on the sign of $(2t+z)$. The reason why both t and z are important is that, in the game with pricing, when a firm increases its own advertising this has two effects on the price its rival charges. First, the increase in a_i directly shifts the rival's demand function, which increases p_j if $t>0$ and decreases it if $t<0$. Second, increasing a_i leads Firm i to increase p_i, which increases demand for Firm j's product if the goods are substitutes (i.e., $z>0$) and decreases it if the goods are complements ($z<0$). The $(2t+z)$ term represents the balance of these two effects in determining whether increasing a_i helps or harms the other firm.

To see this algebraically, recall the condition determining Firm i's optimal price in the third-stage of the pricing game, Eq. 15:

$$p_i = \frac{1}{2}\left(f(a_i) + \mathrm{tf}(a_j) + \mathrm{zp}_j(a_j)\right),$$

where $p_j(a_j)$ captures the fact that p_j depends on a_j. Differentiating with respect to a_j:

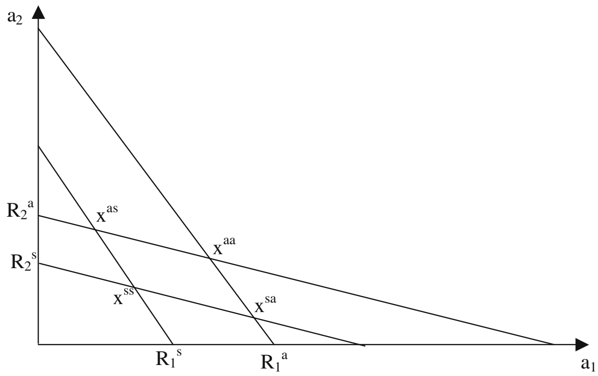
$$\frac{\mathrm{d}p_i}{\mathrm{d}a_j} = \frac{1}{2}\left(\mathrm{tf}'(a_j) + z\frac{\partial p_j}{\partial a_j}\right) = \frac{1}{2}\left(\mathrm{tf}'(a_j) + z\left(\frac{1}{2}f'(a_j)\right)\right) = \frac{1}{2}f'(a_j)\left(t + \frac{z}{2}\right).$$

Thus, an increase in j's advertising increases i's price if and only if the sign of $(2t+z)>0$.[29] And, since equilibrium profit is the square of the equilibrium price, the same condition determines whether increasing advertising helps or harms the other firm. Ultimately, since this drives whether profit increases or decreases along the firm's reaction curves, it is this factor that determines whether setting an advertising budget or sales target dominates in the sales-advertising game.

The strategic nature of the sales-advertising game is slightly different depending on whether $(2t+z)$ is positive or negative. When $(2t+z)$ is positive, then Firm i benefits when the other firm increases its advertising, and consequently it wants to adopt a posture that encourages the other firm to do so. According to Property 1, Firm j will choose a higher advertising level when Firm i sets advertising than when it sets a sales target. Because adopting an advertising strategy induces greater

[29] The fact that t counts twice as much as z arises from the fact that the firm evenly divides the impact on its own demand of increasing its advertising between increasing its price and its quantity.

Fig. 2 The second-stage equilibria with $(2t+z)<0$



advertising expenditure by the other firm, setting an advertising budget is a dominant strategy when $(2t+z)>0$.

On the other hand, when $(2t+z)<0$, then Firm i is harmed when the other firm increases its advertising, and consequently it wants to adopt a strategy that discourages its opponent from doing so. The way to do this is to set a sales-target, since doing so leads the other firm to choose a lower advertising expenditure than setting an advertising budget.

Consider our previous example of the 1997 introduction of MultiCare by P&G and Total by Colgate Palmolive. According to the theory presented above, setting an advertising budget by both firms is optimal as long as $2t+z$ is positive. It is clear that the products are substitutes thus $z>0$. So budget setting is optimal if either advertising for the new products by both firms is category expanding ($t>0$) or more realistically it is rivalrous ($t>0$) but the products are viewed as such close substitutes that it overwhelms the rivalrous effect of advertising.

5 Discussion

In this paper, we have investigated the problem of setting an advertising budget from a competitive standpoint. Rather than taking the environment as given we have instead focused on the strategic implications of setting an advertising budget versus a sales target (with an implicit guarantee of providing as much advertising as necessary to support the desired sales level). As discussed in the introduction, this decision is closely connected to the question of whether, strategically, it is better for a firm to adopt the percentage of sales or objective and task approach to setting advertising budgets. We found that the importance of setting a budget or a sales level by a firm is not so much that it affects the possibilities open to the firm itself, but rather that it influences the other firm's behavior. We show that each firm has a dominant strategy of choosing an advertising budget when its advertising increases the sales of its rival. Conversely, setting a sales level is dominant whenever the rival's sales are decreasing with the advertising. We generalize our results to the case of asymmetric firms as well as allowing for sequential rather than simultaneous announcements by the firms. Finally, we consider three stage interactions where firms not only choose advertising levels but also participate in a differentiated price competition. We show that in this environment the optimal choice of a budget or a sales level crucially depends on whether advertising is expansive or rivalrous as well as the degree of substitutability between the two firms' products.

From the perspective of professional managers and marketers, the main insight from this paper is that the choice of whether to approach competition by setting an advertising budget or a sales target should be driven primarily by strategic considerations. That is, the critical determinant of whether it is better to set an advertising budget or a sales target is how adopting one or the other posture affects the opposing firm's behavior. However, as our models show, the nature of these strategic effects can depend on the nature of the product-market competition.

This dependence is most clearly illustrated in our final model, in which the firms compete through pricing as well as advertising. A typical case is one in which the firms sell substitute products and engage in rivalrous advertising. Here, the net effect of a firm expanding advertising (and thus whether its rival prefers it to be more or less aggressive) depends on the balance between the direct effect of advertising on the opposing firm's demand and the indirect effect (i.e., the fact that when the firm increases advertising it also increases its price, which in turn reduces demand for the opponent's product). However, while this relationship is complex, it is important to note that the relationship is one that firms can estimate using modern marketing techniques and data; the parameters that characterize the balance between these two effects (i.e., t and z) can be estimated using readily available data.

The potential to benefit from the strategic effects resulting from adopting an advertising budget or sales target requires both an understanding of the environment and the ability to credibly adopt one or the other posture. While it may be difficult to commit not to deviate from a pre-specified advertising budget, the uncertainty inherent in competitive environments would seem to make committing to a sales target somewhat more open-ended and, as a consequence, somewhat more difficult to do credibly. While this is true, difficulties in making commitments qualify but do not invalidate our basic results. Some firms will have strong reputations and the financial strength to make it possible to commit to achieving goals in the future "no matter what the cost," while others will not. Firms deciding whether to attempt to gain the strategic advantage of adopting a sales-setting posture should take into account the fact that they should expect their commitment to be only as effective as their ability to convince their rival that they will, indeed, do whatever is necessary to fulfill their commitment to achieve their sales target.

A potential limitation of our analysis is our implicit assumption that the firms can easily invert the market response function to determine the optimal advertising given a sales level target. The inversion clearly depends on the relationship between advertising (prices) and sales being deterministic. When the relationship between advertising and sales has a stochastic component to it, firm's behavior might be more complex. This is because firms will not necessarily be indifferent between holding the advertising budget fixed and achieving a stochastic sales level and committing to a sales level and requiring an ex ante random advertising budget in order to sustain it. We plan to further investigate strategic implications of stochastic relationships between sales and advertising in future work.

Appendix

Proof of Lemma 1 Since the advertising response function is strictly concave its derivative $f'()$ is strictly decreasing, and its inverse, $g(y)$, is also strictly decreasing. Since $|t| < 1$ we have $\frac{1}{1-t^2} > 1$ or $\frac{1}{1-t^2}\frac{1}{p} > \frac{1}{p}$ which implies that $a_F^{aa} > a_F^{ss}$ as well as $a_G^{aa} > a_G^{ss}$, regardless the sign of t. ■

Proof of Lemma 2 Consider Firm 1 in both specifications of the advertising response function:

$$a_{1F}^{aa} - a_{1F}^{as} = a_F^{aa} - \frac{a_F^{ss} - \mathrm{t}a_F^{aa}}{1-t} = \frac{a_F^{aa} - a_F^{ss}}{1-t} > 0$$
$$a_{1F}^{sa} - a_{1F}^{ss} = \frac{a_F^{aa} - \mathrm{t}a_F^{ss}}{1-t} - a_F^{ss} = \frac{a_F^{aa} - a_F^{ss}}{1-t} > 0$$
$$a_{1G}^{aa} - a_{1G}^{as} = a_{1G}^{sa} - a_{1G}^{ss} = a_G^{aa} - a_G^{ss} > 0$$

Where the inequalities are due to Lemma 1. The same is of course true for Firm 2. ■

Proof of Proposition 1 Consider the payoff matrix Eq. 8 and Firm 1

$$\begin{aligned}\Pi_1^{aa} - \Pi_1^{sa} &= \left(\mathrm{pf}\left(a_1^{aa} + \mathrm{t}a_2^{aa}\right) - a_1^{aa}\right) - \left(\mathrm{pf}\left(a_1^{sa} + \mathrm{t}a_2^{sa}\right) - a_1^{sa}\right)\\ &\quad\left(\mathrm{pf}\left((1+t)a^{aa}\right) - a^{aa}\right) - \left(\mathrm{pf}\left((1+t)a^{aa}\right) - \tfrac{a^{aa} - \mathrm{t}a^{ss}}{1-t}\right)\\ &= \tfrac{a^{aa} - \mathrm{t}a^{ss}}{1-t} - a^{aa} = t\,\tfrac{a^{aa} - a^{ss}}{1-t}\end{aligned}$$

$$\begin{aligned}\Pi_1^{as} - \Pi_1^{ss} &= \left(\mathrm{pf}\left(a_1^{as} + \mathrm{t}a_2^{as}\right) - a_1^{as}\right) - \left(\mathrm{pf}\left(a_1^{ss} + \mathrm{t}a_2^{ss}\right) - a_1^{ss}\right)\\ &\quad\left(\mathrm{pf}\left((1+t)a^{ss}\right) - \tfrac{a^{ss} - \mathrm{t}a^{aa}}{1-t}\right) - \left(\mathrm{pf}\left((1+t)a^{ss}\right) - a^{ss}\right)\\ &= a^{ss} - \tfrac{a^{ss} - \mathrm{t}a^{aa}}{1-t} = t\,\tfrac{a^{aa} - a^{ss}}{1-t}\end{aligned}$$

And both of the above differences have the same sign as t. Similarly for Firm 2:

$$\Pi_2^{aa} - \Pi_2^{as} = \Pi_2^{sa} - \Pi_2^{ss} = t\frac{a^{aa} - a^{ss}}{1-t}.$$ ■

Proof of Proposition 2 We start by proving that the dominant strategies lead to higher payoffs than if both firms committed to the other strategy (i.e. if the dominant strategy is for both firms to commit to a sales level then their payoffs are higher than if both firms chose an advertising budget and visa versa.)

Consider the following profit function: $\mathrm{pf}((1+t)a) - a$ it is clearly maximized at a_F^* that satisfies the first order condition[30]:

$$\mathrm{pf}'\left((1+t)a_F^*\right) = \tfrac{1}{1+t}$$
$$a_F^* = \frac{g\left(\frac{1}{1+t}\frac{1}{p}\right)}{1+t}$$

[30] The fact that *f(x)* is bounded guarantees that a^* is finite and strictly positive.

Recall that a_F^{aa} and a_F^{ss} are defined by: $a_F^{aa} = \frac{g\left(\frac{1}{p}\right)}{1+t}$ and $a_F^{ss} = \frac{g\left(\frac{1}{1-t^2}\frac{1}{p}\right)}{1+t}$. If $t>0$ then $\frac{1}{1-t^2} > 1 > \frac{1}{1+t}$ which implies that $g\left(\frac{1}{1+t}\frac{1}{p}\right) > g\left(\frac{1}{p}\right) > g\left(\frac{1}{1-t^2}\frac{1}{p}\right)$ or $\overset{*}{a}_F > a_F^{aa} > a_F^{ss}$. Since $\overset{*}{a}_F$ is the unique maximizer of the strictly concave profit function it is clear that the firms' profits when setting advertising budgets are larger then when they set sales targets. On the other hand, if $t<0$ then $\frac{1}{1+t} > \frac{1}{1-t^2} > 1$ which implies that $g\left(\frac{1}{p}\right) > g\left(\frac{1}{1-t^2}\frac{1}{p}\right) > g\left(\frac{1}{1+t}\frac{1}{p}\right)$ or $a_F^{aa} > a_F^{ss} > \overset{*}{a}_F$. As before it follows that the firms' profit when setting advertising budgets are lower then when they set sales levels. See Fig. 3.

If the advertising response function is $G_i(a_i, a_j) = f(a_i) + t_i f(a_j)$ then we look at $\overset{*}{a}_G = g\left(\frac{1}{1+t}\frac{1}{p}\right)$ the maximizer of $p(1+t)f(a) - a$ and compare it with $a_G^{aa} = g\left(\frac{1}{p}\right)$ and $a_G^{ss} = g\left(\frac{1}{1-t^2}\frac{1}{p}\right)$. We get the exact same t dependent ordering as before.

We now turn to prove (A), when $t < 0, \Pi_F^{ss}$, is the Pareto efficient outcome. By symmetry it is sufficient to deal with the payoffs of one firm say firm 1. We know that $\Pi_{1F}^{ss} > \Pi_{1F}^{aa}$ and from Proposition dominant we get $\Pi_{1F}^{ss} > \Pi_{1F}^{as}$ all we need to show is that $\Pi_{1F}^{ss} > \Pi_{1F}^{sa}$

$$\begin{aligned}\Pi_{1F}^{ss} - \Pi_{1F}^{sa} &= \mathrm{pf}\left((1+t)a_F^{ss}\right) - a_F^{ss} - \left(\mathrm{pf}\left((1+t)a_F^{aa}\right) - \frac{a_F^{aa} - \mathrm{t}a_F^{ss}}{1-t}\right)\\ &= \mathrm{pf}\left((1+t)a_F^{ss}\right) - \mathrm{pf}\left((1+t)a_F^{aa}\right) + \frac{a_F^{aa} - a_F^{ss}}{1-t}\end{aligned}$$

Define $h_1(x) = \mathrm{pf}\left((1+t)a_F^{ss}\right) - \mathrm{pf}((1+t)x) + \frac{x - a_F^{ss}}{1-t}$ clearly $h_1\left(x = a_F^{ss}\right) = 0$ and $h_1'(x) = -p(1+t)f'((1+t)x) + \frac{1}{1-t}$ hence $h'(x)>0$ which implies that $h\left(x = a_F^{aa}\right) > 0$ since $a_F^{aa} > a_F^{ss}$.

Proving (B) is done in a similar way, when $t>0$ we know that $\Pi_{1F}^{aa} > \Pi_{1F}^{ss}$ and $\Pi_{1F}^{aa} > \Pi_{1F}^{sa}$ so if $\Pi_{1F}^{aa} < \Pi_{1F}^{as}$ there is no Pareto efficient outcome and we are done

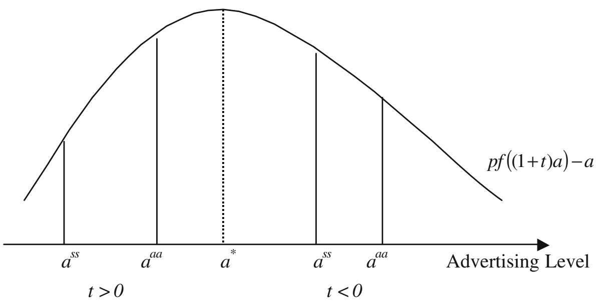
$$\begin{aligned}\Pi_1^{aa} - \Pi_1^{as} &= \mathrm{pf}\left((1+t)a_F^{aa}\right) - a_F^{aa} - \left(\mathrm{pf}\left((1+t)a_F^{ss}\right) - \frac{a_F^{ss} - \mathrm{t}a_F^{aa}}{1-t}\right)\\ &= p\left(f\left((1+t)a_F^{aa}\right) - f\left((1+t)a_F^{ss}\right)\right) + \frac{a_F^{ss} - a_F^{aa}}{1-t}\end{aligned}$$

Consider $h_2(x) = p\left(f((1+t)x) - f\left((1+t)a_F^{ss}\right)\right) + \frac{a_F^{ss} - x}{1-t}$ then $h_2\left(x = a_F^{ss}\right) = 0$ and note that

$$h_2'(x) = p(1+t)f'((1+t)x) - \frac{1}{1-t}$$

which is clearly negative hence $h_2\left(x = a_F^{aa}\right) < 0$ and we get the desired result.

Fig. 3 Profit levels when setting advertising budgets versus setting sales levels



Similar derivation proves the result when using $G_i(a_i,a_j)$ as the response function. ■

Proof of Proposition 4 Substitution of the optimal advertising levels into the two period profit functions and direct comparisons as in the proof of Proposition 1 prove the results. Note that the comparison proves that the strategies are dominant in each period not only for the entire sales season. ■

Proof of Proposition 5 Straightforward calculations show:

$$a_1^{aa} - a_1^{as} = g\left(\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2} - \frac{g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)-t_1g\left(\frac{1}{p}\right)}{1-t_1t_2} = \frac{g\left(\frac{1}{p}\right)-g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)}{1-t_1t_2} < 0$$

$$a_1^{sa} - a_1^{ss} = \frac{g\left(\frac{1}{p}\right)-t_1g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)}{1-t_1t_2} - g\left(\frac{1}{1+t_1t_2}\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2} = \frac{g\left(\frac{1}{p}\right)-g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)}{1-t_1t_2} < 0$$

Where the last inequality is due to the fact that $g(y)$ is decreasing and $\frac{1}{1-t_1t_2} < 1$. Of course the same result holds for Firm 2. ■

Proof of Proposition 6 Consider Firm 1

$$\begin{aligned}\Pi_1^{aa} - \Pi_1^{sa} &= \left(\mathrm{pf}\left(a_1^{aa} + t_1a_2^{aa}\right) - a_1^{aa}\right) - \left(\mathrm{pf}\left(a_1^{sa} + t_1a_2^{sa}\right) - a_1^{sa}\right) \\ &= \left(\mathrm{pf}\left(g\left(\frac{1}{p}\right)\right) - g\left(\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2}\right) - \left(\mathrm{pf}\left(g\left(\frac{1}{p}\right)\right) - \frac{g\left(\frac{1}{p}\right)-t_1g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)}{1-t_1t_2}\right) \\ &= \frac{g\left(\frac{1}{p}\right)-t_1g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)}{1-t_1t_2} - g\left(\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2} = \frac{t_1}{1-t_1t_2}\left(g\left(\frac{1}{p}\right) - g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)\right)\end{aligned}$$

$$\begin{aligned}\Pi_1^{as} - \Pi_1^{ss} &= \left(\mathrm{pf}\left(a_1^{as} + t_1a_2^{as}\right) - a_1^{as}\right) - \left(\mathrm{pf}\left(a_1^{ss} + t_1a_2^{ss}\right) - a_1^{ss}\right) \\ &= \left(\mathrm{pf}\left(g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)\right) - \frac{g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)-t_1g\left(\frac{1}{p}\right)}{1-t_1t_2}\right) \\ &\quad -\left(\mathrm{pf}\left(g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)\right) - g\left(\frac{1}{1+t_1t_2}\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2}\right) \\ &= g\left(\frac{1}{1+t_1t_2}\frac{1}{p}\right)\frac{1-t_1}{1-t_1t_2} - \frac{g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)-t_1g\left(\frac{1}{p}\right)}{1-t_1t_2} = \frac{t_1}{1-t_1t_2}\left(g\left(\frac{1}{p}\right) - g\left(\frac{1}{1-t_1t_2}\frac{1}{p}\right)\right)\end{aligned}$$

Both of the above differences have the opposite sign as t_1 and thus are positive. Similarly for Firm 2 :

$$\Pi_2^{aa} - \Pi_2^{as} = \Pi_2^{sa} - \Pi_2^{ss} = \frac{t_2}{1 - t_1t_2}\left(g\left(\frac{1}{p}\right) - g\left(\frac{1}{1 - t_1t_2}\frac{1}{p}\right)\right)$$

Again these differences are of the opposite sign as t_2 and are thus negative. ■

Proof of Proposition 7

$$\Pi_1^{a\cdot} - \Pi_1^{s\cdot} = \frac{g\left(\frac{1}{1-t^2}\frac{1}{p}\right)}{(1+t)} - \frac{g\left(\frac{1}{1-t^2}\frac{1}{p}\right) - \mathrm{t}g\left(\frac{1}{p}\right)}{(1-t^2)}$$
$$= \frac{t}{1-t^2}\left(g\left(\frac{1}{p}\right) - g\left(\frac{1}{1-t^2}\frac{1}{p}\right)\right)$$

Recall that $g(y)$ is decreasing and $\frac{1}{1-t^2} > 1$ hence the above difference has the same sign as t. ■

Proof of Properties 1–3 in Section 4.3 To show Property 1, recall Firm i's reaction curve when facing an a-setting opponent Eq. 18 and when facing an s-setting opponent, Eq. 21. Both conditions take the form:

$$2\left(\frac{(2+\mathrm{zt})f(a_i) + (2t+z)f(a_j)}{(4-z^2)}\right)\mathrm{k}f'(a_i) = 1,$$

where $k = \frac{(2+\mathrm{zt})}{(4-z^2)}$ in Eq. 18 and $k = \frac{(1-t^2)}{(2+\mathrm{tz})}$ in Eq. 21. Treating a_i as a function of k and differentiating with respect to k yields:

$$\frac{\mathrm{d}a_i}{\mathrm{d}k} = -\frac{((2+\mathrm{tz})f(a_i) + (2t+z)f(a_j))f'(a_i)}{k\left((2+\mathrm{tz})f'(a_i)^2 + ((2+\mathrm{tz})f(a_i) + (2t+z)f(a_j))f''(a_i)\right)}.$$

The denominator is negative by the second-order condition for Firm i's maximization problem,[31] and so $\frac{\mathrm{d}a_i}{\mathrm{d}k} > 0$. Noting that

$$\frac{(2+\mathrm{zt})}{(4-z^2)} - \frac{(1-t^2)}{(2+\mathrm{tz})} = \frac{(2t+z)^2}{(2-z)(z+2)(2+\mathrm{tz})} > 0$$

implies that Firm i's optimal choice of a_i when facing an a-setting opponent is larger than when facing an s-setting opponent.

To prove Property 2, whether Firm i faces an a-setting or s-setting opponent, its optimal choice of a_i satisfies a condition of the form:

$$2\left(\frac{(2+\mathrm{zt})f(a_i) + (2t+z)f(a_j)}{(4-z^2)}\right)\mathrm{k}f'(a_i) = 1.$$

Treating a_i as a function of a_j and differentiating with respect to a_i yields:

$$\frac{\mathrm{d}a_i}{\mathrm{d}a_j} = -\frac{(2t+z)f'(a_j)f'(a_i)}{(2+\mathrm{tz})f'(a_i)^2 + ((2t+z)f(a_j) + (2+\mathrm{tz})f(a_i))f''(a_i)}.$$

[31] This can be easily checked. The details of this and all subsequent computations are available from the authors upon request.

Since the denominator is (once again) negative by the second-order condition, the slope of i's reaction curve has the same sign as $(2t+z)$. Since this expression is independent of k, this is true whether Firm i faces an a-setting or s-setting opponent.

To prove Property 3, consider the case where Firm i faces an a-setting opponent. From its reaction curve Eq. 18:

$$\left((2+\mathrm{zt})f(a_i)+(2t+z)f\left(a_j\right)\right)=\frac{\left(4-z^2\right)^2}{2(2+\mathrm{zt})f'(a_i)}.$$

Firm i's profit is given by:

$$\left(\frac{(2+\mathrm{zt})f(a_i)+(2t+z)f\left(a_j\right)}{4-z^2}\right)^2-a_i.$$

Substituting in the previous expression for Firm i's reaction curve yields the following expression for Firm i's profit for (a_i,a_j) pairs on its reaction curve:

$$\left(\frac{4-z^2}{2(2+\mathrm{zt})f'(a_i)}\right)^2-a_i.$$

Treating a_i as a function of a_j (along Firm i's reaction curve) and implicitly differentiating with respect to a_i yields:

$$\frac{1}{2}\frac{\mathrm{d}a_i}{\mathrm{d}a_j}\left(-2-\frac{\left(4-z^2\right)^2 f''(a_i)}{(2+\mathrm{tz})f'(a_i)}\right)$$

The last factor can be shown to be positive by the second-order condition for Firm i's maximization problem. Hence as a_j increases profit increases if and only if $\frac{\mathrm{d}a_i}{\mathrm{d}a_j}$ is positive, which (by Property 2) occurs whenever $(2t+z)>0$.

A similar computation shows that when Firm i faces an s-setting opponent, profit increases with a_j along its reaction curve if and only if $(2t+z)>0$, which proves Property 3. ■

References

Bass, F. M. (1979). *Advertising spending levels and promotion policies: profit potential for the applications of management science*. The 11th Annual Albert Wesley Frey Lecture, University of Pittsburgh.

Bass, F. M., Krishnamoorthy, A., Prasad, A., & Sethi, S. P. (2005). Generic and brand advertising strategies in a dynamic duopoly. *Marketing Science, 24*, 556–568.

Bulow, J. I., Geanakoplos, J. D., & Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy, 93*, 488–511.

Cheng, L. (1985). Comparing Bertrand and Cournot Equilibria: A geometric approach. *RAND Journal of Economics, 16*, 146–152.

Chintagunta, P., & Jain, D. C. (1995). Empirical analysis of dynamic duopoly model of competition. *Journal of Economics and Management Strategy, 4*, 109–131.

Dube, J. P., Hitsch, G., & Manchanda, P. (2005). An empirical model of advertising dynamics. *Quantitative Marketing and Economics, 3*, 107–144.

Erickson, G. M. (1985). A model of advertising competition. *Journal of Marketing Research, XXII*, 297–304.

Fairhurst, A., Gable, M., & Dickinson, R. (1996) Determining advertising budgets for service enterprises. *Journal of Services Marketing, 10*(6), 18–32.

Farris, P. W., Shames, E. R., & Reibstein, D. J. (1998). *Advertising budgeting: A report from the field*. New York: American Association of Advertising Agencies.
Fudenberg, D., & Tirole, J. (1984). The fat cat effect, the puppy dog ploy, and the lean and hungry look. *American Economic Review, 74*, 361–368.
Hanssens, D. M., Parsons, L., & Schultz, R. L. (2001). *Market response models: Econometric and time series analysis*. Kluwer.
Iyer, G., Soberman, D., & Villas-Boas, J. M. (2005). The targeting of advertising. *Marketing Science, 24*(3), 461–476.
Klemperer, P., & Meyer, M. (1986). Price competition vs. quantity competition: The role of uncertainty. *RAND Journal of Economics, 17*, 618–638.
Little, J. D. C. (1979). Aggregate advertising models: The state of the art. *Operations Research, 27*, 627–667.
Mahajan, V., & Muller, E. (1986). Advertising pulsing policies for generating awareness for new products. *Marketing Science, 5*, 89–106.
Miller, N., & Pazgal, A. (2001).The equivalence of price and quantity competition with delegation. *RAND Journal of Economics, 32*, 284–301.
Miller, N., & Pazgal, A. (2006). Budget or target: The choice between input and output strategies. *RAND Journal of Economics, 37*(2), 391–415.
Mitchell, L. A. (1993). An examination of methods of setting advertising budgets: Practice and the literature. *European Journal of Marketing, 27*, 5–24.
Park, S., & Hahn, M. (1991). Pulsing in a discrete model of advertising competition. *Journal of Marketing Research, XXVIII*, 397–405.
Rao, R. (1986). Estimating continuous time advertising-sales models. *Marketing Science, 5*, 125–142.
Sasieni, M. W. (1971). Optimal advertising expenditure. *Management Science, 18*, 64–72.
Simon, H. (1982). ADPLUS: An advertising model with wearout and pulsation. *Journal of Marketing Research, XIX*, 352–363.
Simon, H., & Arndt, J. (1980). The shape of the advertising response function. *Journal of Advertising Research, 20*, 11–28.
Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics, 15*, 546–554.
Stewart, D. W., & Kamins, M. A. (2002). Marketing communications. In B. A. Weitz, & R. Wensley (Eds.), *The handbook of marketing* (282–309). London: Sage.
Villas-Boas, J. M. (1993). Predicting advertising pulsing policies in an oligopoly: A model and empirical test. *Marketing Science, 12*, 88–102.