Budget or target: the choice between input and output strategies

Nolan H. Miller*

and

Amit I. Pazgal**

In many competitive environments, players need to commit either to a specific goal they will achieve (an output target) or to the resources they are willing to expend in pursuit of that goal (an input budget). We model this situation as a two-stage game where players may compete either by setting input and letting their output follow from the environment ("leading input"), or by setting output and letting the input levels required to support the output targets follow ("leading output"). We show that when each player's output is increasing (decreasing) in his rival's input, leading input (output) dominates leading output (input).

---

*Harvard University; nolan_miller@harvard.edu.

**Washington University in St. Louis; pazgal@wustl.edu.

We thank Steve Anderson, Chris Avery, Brian Mandell, Bill Sandholm, Richard Zeckhauser, the Editor and two anonymous referees, as well as seminar participants at UC Berkeley and UCP for helpful comments.

... This nation should commit itself to achieving the goal, before this decade is out, of landing a man on the moon and returning him safely to the earth.

John F. Kennedy, May 25, 1961.[1]

1 Introduction

With the above statement, President Kennedy shifted the nation's effort in space from "low to high gear."[2] Later that year, Kennedy clarified this statement, saying that the moon mission would involve the considerable expense of developing new rockets, materials, and control systems, but that the U.S. must boldly "do all this, and do it right, and do it first before this decade is out. ...[W]e must pay what needs to be paid."[3] According to Kennedy shortly before his death, "this Nation has tossed its cap over the wall of space, and we have no choice but to follow it. Whatever the difficulties, they will be overcome. Whatever the hazards, they must be guarded against."[4]

Kennedy's position presented a strong challenge to the Soviet Union: the U.S. would do whatever was necessary to win the international prestige associated with being first to the moon. An alternative phrasing of Kennedy's policy would have been to declare that the U.S. would spend $20 billion on the Apollo project during the 1960's, without specifying a final goal. However, the Soviets would have been expected to react differently had Kennedy adopted this approach. After all, dealing with an opponent who is determined to win at all costs is different than dealing with an opponent who states just how far he is willing to go to succeed. In light of this, one might ask whether it was prudent for Kennedy to adopt a goal-oriented (output-setting) rather than a budget-oriented (input-setting) posture.

This paper considers the strategic importance of the choice between input and output strategies, an issue that is not unique to the space race. In a wide range of settings, from fixing an advertising budget to developing a negotiations strategy, participants have the choice between specifying an

[1] "Special Message to Congress on Urgent National Needs." Available from the John F. Kennedy Library at http://www.cs.umb.edu/jfklibrary/j052561.htm.

[2] "Address at Rice University on the Nation's Space Effort," Houston, Texas, September 12, 1962. Available from the John F. Kennedy Library at http://www.cs.umb.edu/jfklibrary/j091262.htm. See Beschloss (1997) for a discussion of Kennedy's decision to go to the moon.

[3] Ibid.

[4] "Remarks at the Dedication of the Aerospace Medical Health Center" San Antonio, Texas, November 21, 1963. Available from the John F. Kennedy library at http://www.cs.umb.edu/jfklibrary/jfk_san_antonio_11-21-63.html.

output strategy – a goal that they will reach at any cost – or an input strategy – the cost that they are willing to incur in pursuit of their goal. Depending on which posture is adopted, the other parties to the relationship will react differently, and this may cause the players to prefer one or the other posture.

We analyze games in which each player's payoff depends on two variables, an input and an output, which respectively correspond roughly to the (dollar-valued) costs and benefits associated with a project. In an R&D contest, the input is the firm's research expenditure, while the output is its expected benefit (i.e., the value of the "prize" times the probability of winning). In oligopolistic competition, the input might be advertising expenditure or investment in cost reduction, while the output is revenue.[5] Each player's goal is to maximize the difference between his output and input.

When there are two players, the structure of the game is captured by four quantities, an input and an output for each player. Each player can approach the game either by setting his input and allowing his output to be determined by the environment and the other player's strategy, or by setting his output and letting his input be determined by the environment and the other player's strategy. Following Jéhiel and Walliser (1995), we call the variable that the player chooses the "leading" variable, and the one that is determined by the environment the "following" variable.

In our basic model we consider two-stage games in which, in the first stage, the players decide whether to lead input or output. After each player has chosen his leading variable, the choices become common knowledge. In the second stage the players compete in a simultaneous-move game, each choosing the specific value of his leading variable.[6] There are four potential second-stage games: both players lead input, both players lead output, and the two mixed cases where one player leads input and the other leads output. We are primarily concerned with the question of whether a player should lead input or output, and so we focus our attention on the "meta-game," the 2x2 game in which the players' strategies are whether to lead input or output and the payoffs are the Nash equilibrium payoffs of the resulting second-stage game.

The main part of the analysis focuses on the case where players are "similar" in the sense that the sign of the effect of increasing player 1's input on player 2's output is the same as the sign of the effect of increasing player 2's input on player 1's output. In this case, the main result is that each

[5] Our use of "input" and "output" is somewhat non-standard. In particular, our use is different than the use in production theory, where inputs include labor and capital and output is measured in units produced.

[6] In order to be clear, throughout the paper we will differentiate between choosing a leading variable (e.g., an advertising budget) and choosing the specific value of a leading variable (e.g., spend $2 million on advertising).

player has a dominant strategy to lead input in the meta-game whenever both players' outputs are increasing in the other player's input. Conversely, leading output is a dominant strategy whenever both players' outputs are decreasing in the other player's input. Because of these dominance relations, the meta-game always has a unique equilibrium.

The results of the paper are driven by the strategic effects of a player's choice of leading variable, i.e., the fact that a player's optimal strategy differs depending on whether he believes his rival to be leading input or output. In particular, we show that, holding fixed the rival player's behavior, a player is indifferent between setting input or setting output. However, when the players are similar, leading output induces a player's rival to behave less aggressively (i.e., use less input) than he would if the player had led input. When the player's output is increasing in his opponent's input, he wants the rival to choose a large input, and leading input encourages his opponent to do so. On the other hand, when the player's output is decreasing in his opponent's input, he wants the rival to choose a small input, and leading output encourages this.

As in any model of strategic interaction, these strategic effects only manifest themselves if the players' commitments are credible, and this credibility may be difficult to achieve. However, it is important to note that our game is one in which the players never act against their own interest. In other words, this game does not have the flavor of an entry deterrence game, where players may threaten to take actions later in the game that are not ex post optimal (e.g., start a price war against a rival who enters the market). In our game, for any choice of leading variables, both players are on their best response curves in the subsequent subgame. Consequently, this is not a game in which commitments are inherently non-credible.

The dominance relations underlying the main results are robust, and we illustrate this by extending the basic model in several directions. We begin by showing that the main results hold in Stackelberg-style games in which the first mover commits to a specific strategy (e.g., achieve $30M in sales) and the second mover then responds optimally to that commitment. In a second extension, we consider the case where committing to output strategies is costly and partial commitments are possible. Although the players' basic incentives persist, in cases where a player prefers to lead output the extent to which he commits to doing so may be limited by the costs involved. Finally, we introduce uncertainty into the model. While the strategic incentives identified in our basic model continue to be important, the stochastic nature of the problem introduces other effects that complicate the analysis. Nevertheless, when uncertainty is not too severe the results on the dominance of leading input or output persist. Even when uncertainty is very severe the strategic effect we identify in our basic model remains, and it must be considered along with these new effects in deciding whether to set input or output.

Our goal in this paper is to illustrate the role of strategic effects in determining whether players' should prefer to lead input or output. Our basic model clearly illustrates the nature of these effects, and the extensions show that the effect persists in more complicated environments. However, real-world strategic interactions are significantly more complicated than our model. In particular, as in the space race, there are important dynamic considerations that we do not consider here. A complete analysis of whether firms should lead input or output in an uncertain, changing environment would require an explicit theory of when and how firms can credibly commit to one or the other leading variable over time. This is a formidable problem in itself, and one that is not directly related to our main focus – analyzing the nature of the strategic interaction when such commitments are possible. Our basic theoretical results are best interpreted as statements regarding what can be achieved when commitments are possible and costless to achieve, the extensions as illustrations of the complications that arise in more realistic models, and our applications as attempts to illustrate situations in which commitments either have been, or could be, effective.

The analysis in this paper is related to the literature on price vs. quantity competition, which considers whether firms competing as oligopolists should choose to compete à la Cournot, setting quantities and letting prices be determined by the market, or à la Bertrand, setting prices and letting market-clearing quantities be determined by the market. For example, Singh and Vives (1984) and Cheng (1985a) show that for a range of reasonable demand structures, it is a dominant strategy for the players to choose to set quantities (prices) when the goods are substitutes (complements).[7] These results rely on two basic ideas: that, holding fixed the rival's price or quantity, firms are indifferent between setting price or quantity, and that firms react more aggressively (i.e., choose lower prices and higher quantities) to a price-setting rival than to a quantity-setting rival. Thus, when the firms sell substitutes and want their rival to be accommodating, they set quantities, whereas when they sell complements and want their rival to be aggressive, they set prices. Although our analysis follows a similar line of argument, our model is distinct, our applications are novel, and we present new technical extensions.

Jéhiel and Walliser (1995) consider generalized duopoly games in which identical players have a

[7] Klemperer and Meyer (1986) consider the impact of uncertainty on whether price- or quantity-setting is superior.

choice between two different control variables. For either player, fixing one of the variables determines the other, and the authors show that, for a broad class of games, one symmetric equilibrium (i.e., where both players choose the same leading variable) dominates the other.

Although related, the generalized duopoly structure of Jéhiel and Walliser (1995) is not particularly helpful in understanding the theoretical question of input versus output setting and the practical applications we have in mind. Unlike in our model, where we differentiate between input and outputs, both of Jéhiel and Walliser's control variables are inputs, which combine (with the other player's variables) to determine the player's utility. Further, the Jéhiel and Walliser analysis assumes that the relationship between the various control variables is linear, which does not seem appropriate for the applications we have in mind.[8]

The paper proceeds as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the game and proves the main result. Section 4 compares the equilibria of the various types of second-stage competition when inputs are strategic complements. Section 5 develops some extensions, Section 6 discusses applications, and Section 7 concludes the body of the paper. Appendix A contains additional technical material and all of the proofs, and Appendix B contains a brief discussion of how the results extend to more general payoff functions.

2 The model

Consider a two-player game in which each player has two control variables, an input and an output. Throughout the paper, denote a generic player by i and the other player by j. Denote player i's input variable by x_i and his output variable by y_i. Given x_i and y_i, player i's payoff is $\pi_i = y_i - x_i$. The most natural interpretation of the variables is that x_i is the input cost incurred by the firm, and y_i is the gross benefit achieved by the firm, both measured in dollars.[9]

The four control variables, x_1, x_2, y_1, and y_2 are related in such a way that if player 1 chooses either x_1 or y_1 and player 2 chooses either x_2 or y_2, these choices determine the values of the remaining two variables. For example, suppose the input variable is advertising expenditure and the output variable is profit from sales. Fixing advertising dollars by each player determines each's profit (assuming a particular pricing rule). Conversely, setting profit targets for each player

[8] In Appendix B, we show that our theory can be extended to subsume Jéhiel and Walliser's analysis.

[9] The results also hold if x_i is measured in physical units and $c_i(x_i)$, and increasing and convex function, measures the dollar cost of using x_i units of input.

determines the advertising levels needed in order to support them. Finally, specifying profit for one player and advertising for the other player determines the two remaining variables.

Formally, for each $i \in \{1, 2\}$ let $y_i(x_i, x_j)$ be player i's output as a function of both players' inputs. Throughout the paper, we confine ourselves to non-negative input vectors that result in non-negative output vectors. We denote the set of such vectors as Ω:

$$\Omega = \{(x_1, x_2, y_1, y_2) : x_1 \geq 0,\ x_2 \geq 0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_1 = y_1(x_1, x_2),\ \text{and}\ y_2 = y_2(x_2, x_1)\}.$$

Since fixing one variable for each player determines the other two variables, we will call a strategy pair consisting of one variable for each player **admissible** if the quadruple it induces is an element of Ω. Strategy pairs that are not admissible induce input-output combinations that are physically impossible given the technology. Hence throughout the paper we restrict players' best responses to be drawn from Ω.

Function $y_i(x_i, x_j)$ gives output when both players choose input as leading. We also define functions that, for any combination of leading variables, give the corresponding following variables. Let $x_i(y_i, y_j)$ be the input player i must provide in order to achieve output level y_i when player j's output is fixed at y_j, obtained by solving identities $y_i(x_i, x_j) \equiv y_i$ for x_i and x_j. Let $\tilde{y}_i(x_i, y_j)$ be the value of y_i that results when player i chooses x_i and player j chooses y_j. Similarly, let $\tilde{x}_i(y_i, x_j)$ be the level of input player i must provide in order to achieve output y_i when player j chooses input x_j. Formally, the functions are related as follows:[10]

$$y_i(x_i(y_i, y_j), x_j(y_j, y_i)) \equiv y_i \qquad x_i(y_i(x_i, x_j), y_j(x_j, x_i)) \equiv x_i$$
$$\tilde{y}_i(x_i(y_i, y_j), y_j) \equiv y_i \qquad \tilde{x}_i(y_i(x_i, x_j), x_j) = x_i.$$

Throughout the paper, we assume that each of $y_i(\cdot,\cdot)$, $x_i(\cdot,\cdot)$, $\tilde{y}_i(\cdot,\cdot)$, and $\tilde{x}_i(\cdot,\cdot)$ is twice differentiable in each of its arguments and strictly increasing in its first argument. We assume the following regularity condition, which, among other things, ensures that $x_i(\cdot,\cdot)$ and $y_i(\cdot,\cdot)$ can both be increasing in their first arguments:

$$\frac{\partial y_i}{\partial x_i}\frac{\partial y_j}{\partial x_j} - \frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i} > 0. \tag{1}$$

[10] Since the technical assumptions necessary to ensure global invertibility of this system impart little economic import to the problem, we will simply assume that the functions are well-defined and that the input-output relationship is invertible. See Cheng (1985b) for the technical assumptions necessary in the case of a system of demand functions.

Assumption (1) amounts to assuming that the "own effects" of increasing one's strategy are larger than the "cross effects". When player i increases his input, this directly increases y_i, but it also affects y_j. If player j has committed to an output target, he must adjust x_j in order to compensate, and this compensation, in turn, affects y_i. If the expression in (1) is negative, then the effect of player j's reaction to the increase in x_i outweighs the direct effect on y_i, and the overall effect of increasing x_i is to *decrease* player i's output. Condition (1) rules out these situations.[11]

In order to ensure that the players' best responses are unique, we assume that $y_i(x_i, x_j)$ is strictly concave in x_i, and that $x_i(y_i, y_j)$ is strictly convex in y_i.[12] Further, since many of the results in the paper are driven by the sign of $\frac{\partial y_i}{\partial x_j}$, we assume that the sign of this partial derivative is independent of the particular input vector at which it is evaluated. That is, either $\frac{\partial y_i}{\partial x_j} > 0$ for all admissible (x_i, x_j) or $\frac{\partial y_i}{\partial x_j} < 0$ for all admissible (x_i, x_j). When $\frac{\partial y_1}{\partial x_2}$ and $\frac{\partial y_2}{\partial x_1}$ have the same sign we call the players **similar**. When they have opposite signs we call the players **dissimilar**.

In the main part of the analysis we consider two-stage games where, in the first stage, each player chooses whether to lead input or output. After the choices have been made, the two leading variables become common knowledge. In the second stage, the players compete by simultaneously choosing the specific values of their leading variables. The equilibrium concept we will employ is subgame perfect Nash equilibrium.

Examples

A number of examples illustrate cases in which these basic assumptions are and are not satisfied.

Example 1: Contests. Consider a situation where two players compete for a prize. The simplest formulation of the problem is that the probability of player i winning the prize is proportional to the ratio of his input to total input, i.e., $y_i(x_i, x_j) = A\frac{x_i}{x_i + x_j}$, where $A > 0$ is the size of the prize. However, since $\frac{\partial y_i}{\partial x_i}\frac{\partial y_j}{\partial x_j} - \frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i} = 0$, this example does not satisfy (1).[13]

A natural contest model that satisfies (1) introduces a small positive probability that neither

[11] Alternatively, when the expression in (1) is negative, the iso-output curve for y_1 is flatter than the iso-output curve for y_2 (when x_1 is plotted on the horizontal axis and x_2 is plotted on the vertical axis). In this case, the situation where both players commit to output targets is inherently unstable. Small deviations from the case where both firms meet their targets drive one firm out of the market (i.e., to choose $x_i = 0$).

[12] Under these conditions, $\tilde{y}_i(x_i, y_j)$ is also strictly concave in x_i.

[13] The reason (1) fails to hold is that knowing y_i determines y_j, since $y_i = A - y_j$, and so it is not possible to invert the system to yield $x_i(y_i, y_j)$ and $x_j(y_j, y_i)$.

party wins the prize. In this case, $y_i(x_i, x_j) = A\frac{x_i}{x_i+x_j+\delta}$, where $\delta > 0$ captures the event that neither party wins. In such a model, if player i chooses output strategy y_i, implying winning probability $\frac{y_i}{A}$, the presence of $\delta > 0$ allows player j to choose an output strategy implying a winning percentage up to (but not including) $1 - \frac{y_i}{A}$. As player j's winning probability increases to $1 - \frac{y_i}{A}$, the input needed to sustain this winning probability increases to infinity.[14]

Example 2: Competition with Investments. Suppose the players are firms that compete in a Cournot oligopoly market with linear demand $p = A - q_1 - q_2$. Firm i's input is an investment that reduce the its production cost. If $c(x_i)$ denotes the firm's cost of producing a unit of output when its input is x_i, output is its profit before deducting investment costs, $y_i(x_i, x_j) = \frac{(A-2c(x_i)+c(x_j))^2}{9}$. This specification satisfies (1) whenever production costs are small enough that the Cournot quantities are non-negative. Output is concave in input provided $c(x_i)$ satisfies an additional regularity assumption. A similar model where the firms produce differentiated products, inputs are advertising expenditures and outputs are sales revenues also satisfies the conditions of our model.[15]

Example 3: Cournot Oligopoly. The standard Cournot oligopoly game can also be thought of in terms of our model. In this game, inputs are expenditures on producing units of the good and outputs are sales revenues. Condition (1) is satisfied whenever the elasticity of demand is uniformly greater than 1 (in absolute value). Thus, for example, our assumptions are satisfied when inverse demand takes the constant elasticity form: $p = A(q_1 + q_2)^{-\frac{1}{\varepsilon}}$ with $A > 0$ and $\varepsilon > 1$. Condition (1) is not generally satisfied for linear demand $p = A - q_1 - q_2$, since demand becomes infinitely elastic as total quantity increases.

3 Equilibrium choice of leading variables

We begin by characterizing the equilibria of the four second-stage games. We then use the results to analyze the subgame-perfect equilibrium of the two-stage game and draw conclusions about the desirability of input-leading and output-leading strategies. The analysis is similar in spirit to Cheng (1985a).

[14] In the event that the players choose strategies with impossible winning probabilities (i.e., $\frac{y_i+y_j}{A} > 1$), it is effectively as if both players have infinite input costs. We therefore restrict the game to input-outut quadruples that imply non-negative inputs and outputs that sum to strictly less than A.

[15] Miller and Pazgal (2005) provides a complete analysis of several versions of the advertising model.

To begin, assume that both players lead input. Player i's optimization problem is to choose x_i to maximize $y_i(x_i, x_j) - x_i$, subject to (x_i, x_j) being admissible. Assuming an interior solution, player i's optimal reaction to x_j is $R_i(x_j)$, implicitly defined by:[16]

$$\frac{\partial y_i(R_i(x_j), x_j)}{\partial x_i} \equiv 1. \tag{2}$$

Reaction function $R_i(x_j)$ is uniquely defined since $y_i(x_i, x_j)$ is strictly concave in x_i. Further, we make the natural assumption that $R_i(0) > 0$. That is, if player j chooses $x_j = 0$, player i finds it worthwhile to produce. When the players compete by setting inputs, the Nash equilibrium input levels are given by x_1^* and x_2^* such that $x_1^* = R_1(x_2^*)$, and $x_2^* = R_2(x_1^*)$.

The following proposition helps to characterize the equilibria of the other types of competition.

Proposition 1 . Player i's best response function depends on whether player j leads input or output, but not on whether player i leads input or output.

The intuition behind Proposition 1 is straightforward. Player i's optimization problem is to choose the value of his leading variable that maximizes his profit, holding fixed the specific value of player j's leading variable. If player i leads input, his output follows; if player i leads output, his input follows. Due to the invertibility assumptions, the resulting input-output pair is the same in either case. Hence what player i is really doing is choosing an input-output pair in response to the value of j's leading variable, and it does not matter which variable leads and which follows. On the other hand, the set of feasible input-output pairs that player i can choose from depends on whether the other player leads input or output, which accounts for the dependence of player i's best response on his rival's choice of leading variable.[17]

In light of Proposition 1, when deriving the equilibria under input leading and output leading we assume that the players always lead input, but do so in response to conjectures that their opponents either lead input or output. Alternatively, one can think of the arguments applying to the projections of the various reaction functions into the input-input space.

We have already considered the case in which player i conjectures that his opponent leads input: his best response function is given by $R_i(x_j)$, as defined in (2). Next, consider the case in which

[16] Throughout the paper, we focus on solutions to the players' problems that are interior to Ω. The results change only slightly if this assumption is relaxed. See Footnote 19.

[17] There is a close analogy between this argument and the discussion of the difference between price and quantity competition. For example, see Singh and Vives (1984), Klemperer and Meyer (1986), and Miller and Pazgal (2001).

player i leads input in response to an output-leading opponent. His optimization problem is to choose x_i to maximize $\tilde{y}_i(x_i, y_j) - x_i$, subject to (x_i, y_j) being admissible. Again assuming an interior solution, differentiating with respect to x_i and setting the result equal to zero yields:

$$\frac{\partial \tilde{y}_i\left(x_i^y\left(y_j\right), y_j\right)}{\partial x_i} \equiv 1,$$

where $x_i^y(y_j)$ is player i's best response to player j's output choice y_j.[18] Let $r_i(x_j)$ be the projection of $x_i^y(y_j)$ into the input-input space: $r_i\left(\tilde{x}_j\left(x_i^y\left(y_j\right), y_j\right)\right) \equiv x_i^y(y_j)$. That is, holding fixed y_j, $x_i^y(y_j)$ implies the same relationship between x_i and x_j as $r_i(x_j)$.

Proposition 2 describes the fundamental relationship between a player's best response to an input-leading opponent and his best response to an output-leading opponent.

Proposition 2 . If the players are similar, player i's optimal input choice is larger when his opponent leads input than when his opponent leads output. That is, $R_i(x_j) > r_i(x_j)$. If the players are dissimilar, the opposite relationship holds: $r_i(x_j) > R_i(x_j)$.[19]

To illustrate the intuition behind Proposition 2, suppose the players are similar, and consider player i. Player i does not directly care about y_j, and he cares about x_j only so much as it affects his own output, y_i. If player i believes player j is holding x_j constant, player i reaps the entire gain from increasing x_i. On the other hand, if player j is holding his output fixed, then player i conjectures that player j will match any change in x_i with the corresponding change in x_j necessary to keep y_j constant. For example, if increasing x_i increases y_j, then player j will respond to any increase in x_i by decreasing x_j, which will in turn tend to decrease y_i (by similarity). Hence player j's output-setting behavior dampens the effect on y_i of an increase in x_i. Since the marginal benefit of increasing x_i is now lower, player i is less inclined to increase his input, and his reaction to output-setting is smaller than his reaction to input-setting. Figure 1, Panel A, depicts sample reaction functions when players are similar. By Proposition 2, r_i lies everywhere below of R_i.

When players are dissimilar, the opposite reasoning applies (see Figure 1, Panel B). When player i leads input against an output-leading rival, player i anticipates that player j will respond

[18] Convexity of $x_i(y_i, y_j)$ in y_i combined with the invertibility assumptions ensure that $x_i^y(y_j)$ is uniquely defined for each y_j. This can be confirmed by differentiating $\tilde{y}_i(x_i(y_i, y_j), y_j) \equiv y_i$ twice with respect to y_i.

[19] If we allow for the possibility of non-interior solutions to the players' optimization problems, the inequalities in Proposition 2 would be weak instead of strict, and the remainder of the results would follow with only slight modifications.

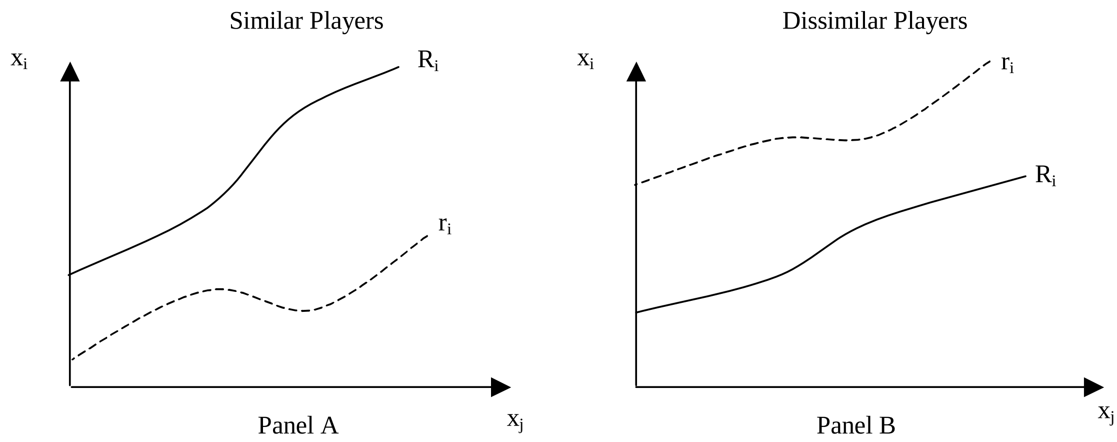

Figure 1: Similar and dissimilar players' reactions to input- and output-leading rivals.

to any increase in x_i by adjusting his input in order to keep his output constant. But, unlike in the case of similar players, with dissimilar players this adjustment *benefits* player i. Thus, player j's compensation to maintain his output target increases the marginal benefit of an increase in input by player i, which encourages player i to choose larger inputs than if he faced an input-leading opponent.

Beyond insisting that they be single-valued when viewed as functions of x_j, none of the conditions imposed thus far restrict the shape of R_i or r_i in any way.[20] In the following section, we impose conditions on the cross-partial derivatives of y_i that restrict the shape of the reaction functions and exploit these restrictions in order to draw conclusions about the relative magnitudes of equilibrium inputs, outputs, and payoffs in the various specifications of the game. These conditions are not needed to determine the equilibria, however, only to compare them.

In the second stage of the two-stage game there are four subgames to consider: both players lead input (xx), both players lead output (yy), player 1 leads input and 2 leads output (xy), and player 1 leads output and player 2 leads input (yx).[21] The equilibria of these subgames are given

[20] Although we tend to draw R_i and r_i as generally upward sloping, we do so for clarity of the diagrams rather than because this shape is required by the model.

[21] Throughout the paper, we assume that in each of the four subgames the equilibrium is unique. One sufficient condition for this to hold would be that the players' best response functions are contractions. See, for example, Vives (1999). Although the analysis is greatly simplified if there is a unique equilibrium in each second-stage game, the analysis is unaffected by the possibility of multiple equilibria in some subgames. See, for example, Cheng (1985a) for a discussion of this point in the context of the choice of prices or quantities as leading variables in a

by the intersection of R_1 and R_2, r_1 and r_2, r_1 and R_2, and R_1 and r_2, respectively, as depicted in Figure 2, Panel A, for the case of similar players.[22] Panel B of Figure 2 depicts the case of dissimilar players.

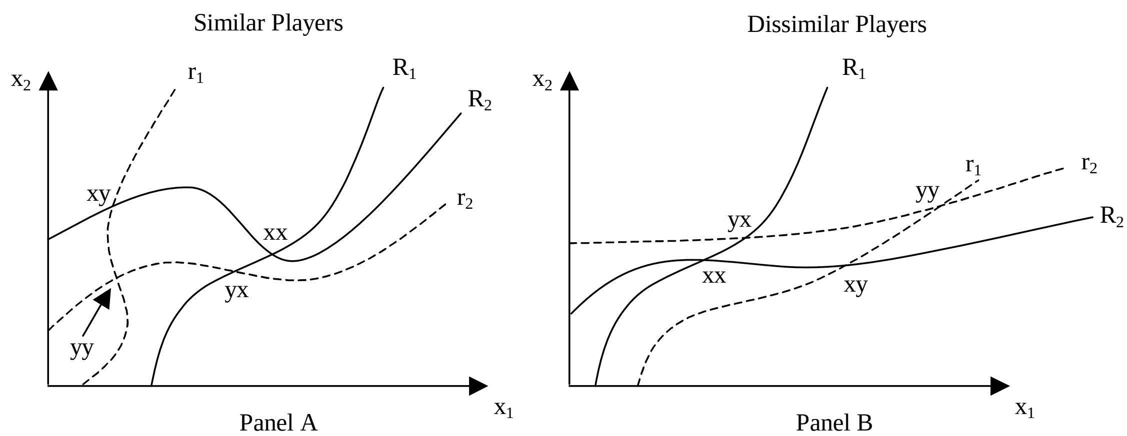

Figure 2: Equilibria of the four subgames for similar and dissimilar players.

The two-stage game in which players choose leading variables and then choose the specific values of their leading variables can be reduced to a 2x2 normal-form game – the "meta-game" – the equilibrium of which is the subgame perfect equilibrium of the two-stage game. The payoffs in the meta-game are given by the players' payoffs at points xx, yy, xy, and yx in Figure 2.

Denote the equilibrium input vector when player 1 chooses $s \in \{x, y\}$ as leading and player 2 chooses $t \in \{x, y\}$ as leading as $\left(x_1^{st}, x_2^{st}\right)$. Let $\pi_i\left(x_i, x_j\right) = y_i\left(x_i, x_j\right) - x_i$ be player i's profit as a function of the input vector, and let $\pi_i^{st} = \pi_i\left(x_i^{st}, x_j^{st}\right)$ be player i's equilibrium payoff when 1 leads with s and 2 leads with t. Table 1 depicts the normal form of the meta-game.

differentiated-products duopoly.

[22] Recall that r_1 is player 1's reaction curve to an output-leading opponent, and not player 1's reaction curve when he, himself, leads output.

		Player 2	
		Lead x_2	Lead y_2
Player 1	Lead x_1	(π_1^{xx}, π_2^{xx})	(π_1^{xy}, π_2^{xy})
	Lead y_1	(π_1^{yx}, π_2^{yx})	(π_1^{yy}, π_2^{yy})

Table 1: The normal form of the meta-game.

The following Lemma establishes that π_i changes monotonically along R_i and r_i.

Lemma 1 . The signs of $\frac{\partial \pi_i(R_i(x_j),x_j)}{\partial x_j}$ and $\frac{\partial \pi_i(r_i(x_j),x_j)}{\partial x_j}$ are the same as the sign of $\frac{\partial y_i}{\partial x_j}$. Hence if $\frac{\partial y_i}{\partial x_j} > 0$, π_i increases along both $R_i(x_j)$ and $r_i(x_j)$ as x_j increases, and if $\frac{\partial y_i}{\partial x_j} < 0$, π_i decreases along both $R_i(x_j)$ and $r_i(x_j)$ as x_j increases.

Lemma 1 holds regardless of whether firms are similar or dissimilar. It depends only on the sign of $\frac{\partial y_i}{\partial x_j}$ remaining the same for all points on the relevant reaction curves. Lemma 1 implies that each player has a dominant strategy in the meta-game.

Proposition 3 . For similar players, leading input is a dominant strategy in the meta-game for player i if $\frac{\partial y_i}{\partial x_j} > 0$. Leading output is dominant for player i if $\frac{\partial y_i}{\partial x_j} < 0$. For dissimilar players, the opposite relations hold: leading input is dominant if $\frac{\partial y_i}{\partial x_j} < 0$ and leading output is dominant if $\frac{\partial y_i}{\partial x_j} > 0$.

Suppose player 2 leads input. If player 1 switches from leading input to leading output, the outcome of the game moves along R_1 from xx to yx. If player 2 sets output and player 1 switches from leading input to leading output, the outcome moves along r_1 from xy to yy. Proposition 3 implies that $\pi_1^{yx} - \pi_1^{xx}$ and $\pi_1^{yy} - \pi_1^{xy}$ have the same sign, and therefore that player 1 has a dominant strategy in the meta-game. The same argument applies to player 2, and thus the meta-game has a unique equilibrium, as described in Proposition 4.

Proposition 4 . When players are similar, the unique subgame-perfect Nash equilibrium of the meta-game involves both players leading input if $\frac{\partial y_i}{\partial x_j} > 0$ and both players leading output if

$\frac{\partial y_i}{\partial x_j} < 0$. If players are dissimilar, then the unique equilibrium of the meta-game involves the player for whom $\frac{\partial y_i}{\partial x_j} > 0$ leading output and the player for whom $\frac{\partial y_i}{\partial x_j} < 0$ leading input.

Proposition 4 is the main result of this paper. Holding fixed the leading variable of player j (but not its specific level), by changing which variable he leads, player i can force player j to increase or decrease his input usage in the second stage. Since changing leading variables has no direct effect on player i's behavior (Proposition 1) but potentially beneficial strategic effects on player j's behavior (Proposition 2), player i will exploit this ability in his choice of leading variable. Since player i's preferences over leading variables do not depend on his opponent's leading variable, he has a dominant strategy in the meta-game (Proposition 3), and hence (since the same reasoning implies that player j also has a dominant meta-game strategy) the unique equilibrium of the meta-game involves each player playing his dominant strategy (Proposition 4).

The intuition underlying the meta-game equilibrium is slightly different depending on whether the players are similar or not and on the signs of $\frac{\partial y_1}{\partial x_2}$ and $\frac{\partial y_2}{\partial x_1}$. Consider similar players. When $\frac{\partial y_i}{\partial x_j} > 0$ the meta-game involves a free-rider problem. Each player is tempted to choose a small input value, relying on the positive external effects of the other player's input to increase his output. Leading output exacerbates the problem: by doing so the player announces his intention to respond to any increase in x_j by decreasing x_i in order to keep output y_i constant. And, since this reaction harms player j, player j is consequently less willing to increase his input. The result is that both players choose relatively small inputs. On the other hand, leading input mitigates the free-rider problem. By leading input, player i commits to expend a certain amount regardless of player j's input choice, and so player j no longer fears that an increase x_j will be counteracted by a decrease in x_i. Consequently, player j is willing to choose a higher input, both players will choose relatively large inputs in equilibrium, and the magnitude of the free-rider problem is reduced.

Games with similar players and $\frac{\partial y_i}{\partial x_j} < 0$ are particularly important. We call such games **games of strictly opposed outputs**, since the effects of a change in input on a player's own output and on his rival's output have opposite signs. The three examples of Section 2 all fall into this category, as does any game with the flavor of a contest where the players compete for a prize but each bears his own cost of the effort needed to win it (e.g., a negotiation). When the players lead inputs in such a game, any increase in input by player i harms player j, and therefore gives player j an incentive to increase x_j in order to compensate. The result is that each player has an incentive to choose x_i large in order to prevent his opponent from gaining an advantage, leading to an "input war." In this case, leading output can help to alleviate the problem. By leading output, player i announces to player j that he will meet any increase in x_j with the increase x_i necessary to maintain y_i. Since this compensation is harmful to player j, player j will consequently be less willing to increase his own input. And, relieved of the fear the player j is going to choose a large input, player i is willing to also choose a smaller input as well. Thus leading output is dominant in games of strictly opposed outputs.

When players are dissimilar, the intuition behind Proposition 4 remains the same, although in this case players are pushed toward the asymmetric outcomes of the meta-game. Recall that with dissimilar players leading input encourages the other player to use more inputs than leading output, and suppose, for example, that $\frac{\partial y_1}{\partial x_2} > 0$ and $\frac{\partial y_2}{\partial x_1} < 0$. In this case, player 1 would like player 2 to choose x_2 large. Since $r_2 > R_2$, adopting output as leading encourages this. On the other hand, player 2 would like player 1 to choose x_1 small, and leading input encourages this. Thus, the equilibrium of the meta-game would be at point (yx) in Figure 2, Panel B.

Our basic analysis considers players whose preferences are given by $y_i - x_i$ because the additive representation of preferences is the most natural one in many of the applications we consider. However, the techniques employed to prove the results in this section extend in a straightforward way to more general preferences. In particular, the results continue to hold whenever each player's preferences over output and input are quasiconcave. In Appendix B we briefly sketch how such a generalization can be incorporated into our analysis and show that, in this extended framework, our analysis subsumes the results of Jéhiel and Walliser (1995).

4 Comparing the outcomes of the meta-game

Based on the results of the previous section, no conclusions can be drawn about how the usage of inputs and production of outputs compares across the four subgames because we have as yet done nothing to restrict the shape of the players' reaction curves. In this section, we show that if inputs are strategic complements and players are similar, much can be said about how the outcomes of the meta-game compare. However, if players are dissimilar or inputs are not strategic complements, less can be said without imposing additional restrictions on the input-output relationship.

To begin, focus on similar players. Differentiating equation (2) with respect to x_j, the slope of

$R_i(x_j)$ is given by:

$$\frac{dR_i(x_j)}{dx_j} = -\frac{\frac{\partial^2 y_i}{\partial x_i \partial x_j}}{\frac{\partial^2 y_i}{\partial x_i^2}}. \tag{3}$$

The denominator of the right hand side of (3) is negative by concavity. Hence the slope of the reaction function has the same sign as the cross-partial derivative, $\frac{\partial^2 y_i}{\partial x_i \partial x_j}$. If inputs are strategic complements $\left(\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0\right)$ for all admissible (x_i, x_j), then $R_i(x_j)$ is increasing in x_j. If inputs are strategic substitutes $\left(\frac{\partial^2 y_i}{\partial x_i \partial x_j} < 0\right)$ for all admissible (x_i, x_j), then $R_i(x_j)$ is decreasing.

When inputs are strategic complements, we can make the following comparison.

Proposition 5 . If players are similar and $\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0$, then the symmetric outcomes of the meta-game compare as follows:

i) If $\frac{\partial y_i}{\partial x_j} > 0$, then input leading both players results in higher inputs, outputs, and profits than output leading by both players. The outcome when both players lead input Pareto dominates the other outcomes of the meta-game.

ii) If $\frac{\partial y_i}{\partial x_j} < 0$, then input leading by both players results in higher inputs than output leading by both players. When both players lead output, player i earns a higher payoff than when either both players lead input or player i leads input and player j leads output, but may earn a lower profit than when player i leads output and player j leads input.

Since input-leading is the equilibrium of the meta-game when $\frac{\partial y_i}{\partial x_j} > 0$, the equilibrium of the meta-game Pareto dominates its other outcomes. When $\frac{\partial y_i}{\partial x_j} < 0$, the equilibrium of the meta-game Pareto dominates the other symmetric outcome of the game, but may not dominate the asymmetric outcomes.

As is usually the case, strategic complementarity adds a great deal of structure to the model, allowing definite statements to be made. Less can be said about the case where inputs are strategic substitutes. To see why, consider Figure 3, which illustrates a possible configuration of the outcomes of the meta-game. As drawn, yy involves a larger value of x_1 than xx. Because this possibility cannot be ruled out, no analogue of Proposition 5 exists when inputs are strategic substitutes. It can, however, be said that at least one player employs less input at yy than at xx, and that if the players are symmetric, i.e., $y_1(a,b) = y_2(b,a)$, both do.

If the sign of $\frac{\partial^2 x_i}{\partial y_j \partial y_i}$ is known, it may be possible to further describe the meta-game equilibrium. In particular, there would be a "dual" result for Proposition 5. However, there is no necessary

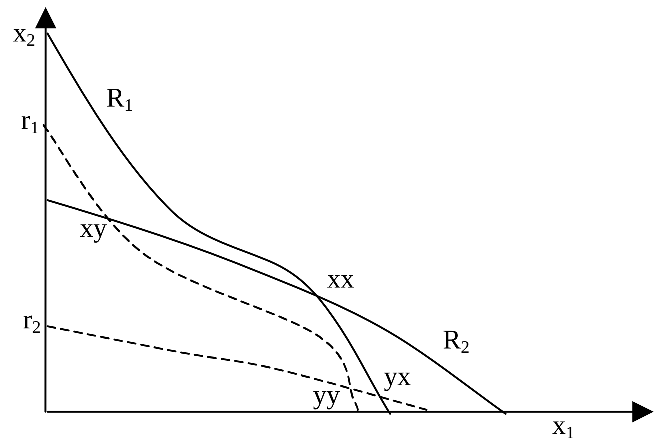

Figure 3: Equilibria when inputs are strategic substitutes.

relationship between $\frac{\partial^2 x_i}{\partial y_j \partial y_i}$ and $\frac{\partial^2 y_i}{\partial x_i \partial x_j}$. In particular, the relationship depends on the first and second derivatives of $y_i(x_i, x_j)$ with respect to x_j, the signs of which are not determined in the model. An analogue to Proposition 5 does exist when players are dissimilar and inputs are strategic substitutes. However, the comparisons are between the asymmetric outcomes, xy and yx.

5 Extensions

In this section we provide three extensions to the basic analysis. First, we consider a sequential-move game where, instead of merely announcing whether he will be a budget-setter or target-setter, the first-mover begins the game by announcing a particular budget or target. Second, we consider the case where commitment to a target is costly and partial commitments are possible. Finally, we examine a simple stochastic version of the model to illustrate the additional considerations that may arise in such a setting. In each case, while the analysis becomes more complicated, the basic strategic effect identified in the non-stochastic case continues to be important.

A sequential-move game

In the preceding analysis, the player's first-stage choice is between being an "input-setter" or an "output-setter". However, real-world competitions rarely begin with a player declaring "I will be an output-setter." More often, as occurred with the race to the moon, a player declares a specific output target (or input budget). In this section, we extend the analysis to games of this sort.

We consider a "Stackelberg" version of the basic game in which player 1 moves first and player 2 moves second. As first-mover, player 1 credibly commits not only to being an input-setter or an output-setter, but also to a particular value of his input or output. After observing player 1's strategy choice, player 2 moves second.

Formally, the game can be written as:

Stage 1: Player 1 chooses $s_1 \in S_1 = X_1 \cup Y_1$, where $X_1 \subset \mathbb{R}_+$ is the set of feasible input strategies, and $Y_1 \subset \mathbb{R}_+$ is the set of feasible output strategies.

Stage 2: After observing s_1, player 2 chooses x_2 in order to maximize his payoff subject to the constraint that (s_1, x_2) is admissible, where player 2's payoff is given by:[23]

$$\pi_2 = \begin{cases} y_2(x_2, s_1) - x_2 & \text{if } s_1 \in X_1, \text{ and} \\ \tilde{y}_2(x_2, s_1) - x_2 & \text{if } s_1 \in Y_1. \end{cases}$$

A strategy for player 2 is a function, $x_2(s_1)$, which specifies the input player 2 chooses in response to any strategy choice by player 1.

Stage 3: Payoffs are realized as determined by strategy choices s_1 and $x_2(s_1)$.

Stage 3 is a production stage in which player 1 is committed to use input s_1 if $s_1 \in X_1$ or produce output $s_1 \in Y_1$. We leave the commitment mechanism a "black box" except to note that, as in all of our games, to the extent that the firms are unable to commit, they will be unable to influence their opponent's reactions and thereby gain a strategic advantage.

This game has a unique subgame perfect equilibrium (SPE), which can be found by backward induction. Proposition 6 extends the basic analysis to this case. The intuition behind the result is the same as in the basic model.

Proposition 6 . Suppose players are similar. If $\frac{\partial y_1}{\partial x_2} > 0$, then in the SPE of this game, player 1 chooses an input, $s_1 \in X_1$. If $\frac{\partial y_1}{\partial x_2} < 0$, then in the SPE of this game, player 1 chooses an output, $s_1 \in Y_1$. The opposite conclusions hold if players are dissimilar.

Commitment costs and partial commitment

Until now, we have assumed that players are able to costlessly commit to being input-setters or output-setters. Here, we relax that assumption.

[23] Since Proposition 1 still applies, without loss of generality we assume that player 2 chooses input x_2.

To begin considering the issue of credible commitment, it is important to note that there is no private information in the game we consider. Inputs and outputs are observable and verifiable. In such environments there are a number of mechanisms that can be used to augment the credibility of a player's commitments and to communicate them to his rival, some of which are discussed by Schelling (1960) and Dixit and Nalebuff (1991). For example, if the players engage in many interactions like the game we consider, then reputational concerns may help make their commitments credible. Also, since inputs and outputs are verifiable, players may enter into contracts with suppliers or customers that increase the punishment from deviating from their commitments. Finally, the players may delegate their decision-making power to agents who are given incentives to adopt the appropriate leading variable.

While such mechanisms may help the player to credibly commit, there may be costs associated with doing so, and total commitment/credibility may not be possible. In such cases, we would expect that the players would weigh the costs and benefits of commitment in deciding on their strategies. In this section we provide a brief illustration of this point.

We might expect that it is more costly to commit to setting output than to setting input since outputs such as revenue seem inherently more dependent on the environment and the other player's choices than input expenditures. We adopt this assumption in our model, although the analysis would be similar if it were more costly to be an input-setter than an output-setter. We consider the impact on the results of imposing a cost on committing to be an output setter, and we also allow for players to make partial commitments. To illustrate the role of costly commitment, we focus on player 1's incentives and assume that player 2 sets input.

We model partial commitment to being an output-setter by p, the probability that player 2 assigns to player 1 behaving as an output-setter ($0 \leq p \leq 1$).[24] We model the cost of committing to behaving as an output setter by $c(p)$, with $c(0) = 0$, $c'(p) > 0$ and $c''(p) > 0$.

The game consists of two stages. In the first stage, player 1 chooses p, and this choice is observed by player 2. In the second stage, the players simultaneously choose their strategies.

We begin by studying the second-stage equilibrium, conditional on a particular choice of p. Since player 2 sets input, player 1's reaction curve is fixed and defined implicitly by (2). Player 2, on

[24] This approach is similar to the conjectural variations approach to oligopoly games. See Vives (1999) for a discussion of the conjectural variations approach in the Cournot vs. Bertrand literature, and Miller and Pazgal (2001) for an examination of strategic effects in duopoly games when a wider range of behaviors is permitted.

the other hand, chooses his input in order to maximize $(1-p)\left[y_2\left(x_1, x_2\right)-x_2\right]+p\left[\tilde{y}_2\left(y_1, x_2\right)-x_2\right]$, subject to admissibility. Omitting the derivatives' arguments for clarity, the first-order condition for an interior solution to this problem is $(1-p) \frac{\partial y_2}{\partial x_2}+p \frac{\partial \tilde{y}_2}{\partial x_2}=1$, or, making use of the fact that $\frac{\partial \tilde{y}_2}{\partial x_2}=\frac{\partial y_2}{\partial x_2}-\left(\frac{\partial y_2}{\partial x_1} \frac{\partial y_1}{\partial x_2}\right) / \frac{\partial y_1}{\partial x_1}$ by Observation 3 in Appendix A (see equation (A3)):

$$\frac{\partial y_2}{\partial x_2}-p \frac{\frac{\partial y_2}{\partial x_1} \frac{\partial y_1}{\partial x_2}}{\frac{\partial y_1}{\partial x_1}}=1.$$

Note that when $p=0$, player 2 believes player 1 leads input and when $p=1$, player 2 believes player 1 leads output. As p increases from 0 to 1, player 2's reaction shifts inward from $R_2\left(x_1\right)$ to $r_2\left(x_1\right)$, and the second-stage equilibrium moves along player 1's reaction curve, R_1. This inward movement increases player 1's profit if $\frac{\partial y_1}{\partial x_2}<0$ and decreases it if $\frac{\partial y_1}{\partial x_2}>0$.

Thus, in the case of costly/partial commitment, the direction of player 1's incentives remains the same. If increasing x_2 increases his profit, player 1 wants x_2 to be large, and so prefers to lead input. If increasing x_2 decreases his profit, player 1 wants x_2 to be small and so prefers to lead output. What changes is that in this case there is a cost associated with adopting an output-setting posture. Thus, as player 1 considers whether to undertake actions that increase his commitment to output-setting behavior, he must ask himself whether the benefit from doing so outweighs the cost. Let $x_1(p)$ and $x_2(p)$ denote the second-stage equilibrium inputs as a function of p. Player 1 chooses $p \in[0,1]$ to maximize $y_1\left(x_1(p), x_2(p)\right)-x_1(p)-c(p)$, which (omitting the derivatives' arguments for clarity) yields first-order condition for an interior solution p^*:

$$\frac{\partial y_1}{\partial x_1} \frac{d x_1}{d p}+\frac{\partial y_1}{\partial x_2} \frac{d x_2}{d p}-\frac{d x_1}{d p}=c^{\prime}\left(p^*\right).$$

When $\frac{\partial y_1}{\partial x_2}>0$, player 1 prefers to set input even when commitment is costless, and so he still prefers to set input with the commitment cost, and chooses $p=0$. When $\frac{\partial y_1}{\partial x_2}<0$, player 1 would choose $p=1$ if commitment were costless. When commitment is costly, he sets p so that the marginal benefit from increasing his commitment to output-setting equals the marginal cost of doing so.

Uncertainty

In our basic model, the relationship between inputs and outputs is deterministic. However, many real-world conflicts have an inherently stochastic nature. In this section we provide a simple stochastic version of our model that illustrates the impact of uncertainty on the results.

The presence of uncertainty gives rise to several new effects that capture how the players adjust their strategies in the face of uncertainty, and this adjustment can either mitigate or reinforce the non-stochastic strategic effect. Although they affect the nature of the strategic interaction between the parties, these effects are not essentially strategic in nature; they would be present even in a monopolistic model in which the firm's marginal product (or cost) function has a random component.

Three key insights emerge from the analysis of the stochastic model. First, the basic strategic effect we identified in the non-stochastic model (which we will refer to as the non-stochastic strategic effect, or NSE) remains in stochastic environments. However, the players' overall incentives are determined by this non-stochastic strategic effect as well as the players' desire to adjust their strategies in light of the uncertainty. Second, because the NSE remains, the basic results on the dominance of input-setting or output-setting are robust to the introduction of a moderate amount of uncertainty. Third, if the effects of the uncertainty are sufficiently large, this could upset the results on whether input-setting or output-setting is preferred. Nevertheless, even in such cases the non-stochastic strategic effect remains, and it must be weighed against the uncertainty effects in determining the players' optimal strategies.

Our analysis of the non-stochastic model made use of the fact that setting any two of the four strategic variables uniquely determines the other two. This permitted us to project the players' reaction curves from the output-output and mixed games into the input-input space, which greatly facilitated the analysis. In the presence of uncertainty, this is no longer possible. To see why, suppose that both players set outputs. Holding fixed outputs, uncertainty about the firms' production functions implies the inputs needed to sustain those outputs become stochastic. Thus specifying pairs of outputs implies (non-degenerate) *distributions* over inputs. Because of this, the general arguments made in the non-stochastic case are no longer possible.

We begin by considering a particularly simple stochastic setting and focus on the incentives faced by one player, in this case player 1. Player 2 is assumed to lead input. The game begins with player 1 choosing and announcing his leading variable. Following this announcement, the players choose their strategies. Next, a stochastic shock to player 1's output takes place. Finally, the two players' inputs and outputs are determined. To keep things as simple as possible while illustrating the effects of uncertainty, we assume that player 2's production function is non-stochastic and that the players are symmetric.

Player 1's stochastic output function, which we denote $Y_1(x_1, x_2, \tilde{z})$, takes the form $Y_1(x_1, x_2, z) = y_1(x_1, x_2) + z$, where $y_1(x_1, x_2)$ is the output production function we considered earlier and z is the realization of a zero-mean random variable, $\tilde{z}$. When player 1 leads output in this game, he commits to a total output Y_1. As a result, the *production* needed in order to achieve this target is stochastic: $Y_1 - \tilde{z}$ units must be produced in order to meet the target. Thus, one possible interpretation of the game is that this is the last period of a multi-period production game, and the realization of $\tilde{z}$ captures how well player 1 has done in the game until now (which is unknown at the time he must set his production plan). If $\tilde{z} < 0$, times have been bad and if player 1 has committed to an output strategy he must make up this deficit in the final period. Conversely, if $\tilde{z} > 0$, times have been good, and player 1 can meet his output target without producing as much in the final period as he had planned.

Before considering the strategic effects of adopting an input-setting or output-setting posture, we first show that there is a direct advantage to leading input in this game.[25]

Proposition 7 . Holding fixed player 2's input choice in the game described above, player 1's maximal profit is larger when he sets input than when he sets output.

When player 1 sets input, his output is stochastic. However, given the additive nature of the uncertainty this has no effect on the player's expected profit. On the other hand, when player 1 leads output his total output is fixed, but the output he needs to *produce* (and hence how much cost he must incur) is stochastic. Due to the convexity of the player's cost function, the expected cost of producing $Y_1 - \tilde{z}$ units is larger than the cost of producing the expected output, Y_1. Because of this, maintaining an output target in the presence of uncertainty is more costly to the firm than maintaining the input target that implies the same expected output.[26] We call this effect, which biases player 1 toward an input-setting posture, the Own Uncertainty Effect (OUE).

[25] Klemperer and Meyer (1986) prove a similar result in the context of the Cournot vs. Bertrand debate. In their case, they show that whether uncertainty biases a player toward price- or quantity-setting depends on the nature of the uncertainty. Although the specification we work with is a natural one, there are other models in which uncertainty would bias the player toward output-setting.

[26] While we have argued that input-setting is preferred to output setting holding fixed the other firm's strategy, we cannot say whether uncertainty tends to increase or decrease the firm's output target. The reason is that this depends on comparing $\frac{\partial \tilde{x}_1}{\partial y_1}$ and $E_{\tilde{z}} \frac{\partial \tilde{x}_1}{\partial y_1}$, which in turn depends on the whether $\frac{\partial \tilde{x}_1}{\partial y_1}$ is concave or convex in y_1. Reasonable general restrictions on the players' production functions do not determine the concavity of convexity of $\frac{\partial \tilde{x}_1}{\partial y_1}$, but it is easily determined in particular cases which (if either) of these is satisfied.

While the OUE biases player 1 toward choosing to set input *holding fixed player 2's strategy*, choosing to lead input or output also influences player 2's strategy choice. To gain an understanding of how the introduction of uncertainty affects the overall strategic effect of choosing to lead input or output, consider player 2's optimal reaction to an input-leading or output-leading opponent. If player 1 sets input, then player 2 chooses x_2 to maximize $y_2(x_1, x_2) - x_2$ subject to (x_1, x_2) being admissible, which for an interior solution yields first-order condition $\partial y_2(x_1, x_2^*)/\partial x_2 = 1$. Note that this is the same first-order condition as in the non-stochastic case.

If player 1 sets output Y_1, then player 2 sets x_2 in order to maximize $E_{\tilde{z}}(\tilde{y}_2(Y_1 - z, x_2) - x_2)$ subject to $(Y_1 - \tilde{z}, x_2)$ being admissible for all realizations of $\tilde{z}$, which for an interior solution has optimality condition:

$$E_{\tilde{z}} \frac{\partial \tilde{y}_2(Y_1 - z, x_2^*)}{\partial x_2} = 1.$$

Here, player 2 equates his expected marginal product, which depends on player 1's stochastic production $Y_1 - \tilde{z}$, with his marginal cost.

Whether player 2 chooses a larger input in response to input-setting or output-setting (i.e., the direction of the strategic effect) depends on the relative size of $\frac{\partial y_2(x_1, x_2^*)}{\partial x_2}$ and $E_{\tilde{z}} \frac{\partial \tilde{y}_2(Y_1 - z, x_2^*)}{\partial x_2}$, where $Y_1 = y_1(x_1, x_2^*)$ is the expected output when inputs are x_1 and x_2^*. To facilitate this comparison, we decompose their difference, which we call the total strategic effect (TSE), into the non-stochastic strategic effect (NSE) and the rival's reaction to his uncertainty about the player's strategy, which we call the stochastic strategic effect (SSE):

$$\overbrace{\frac{\partial y_2(x_1^*, x_2^*)}{\partial x_2} - E_{\tilde{z}} \frac{\partial \tilde{y}_2(Y_1 - z, x_2)}{\partial x_2}}^{\text{TSE}}$$
$$= \overbrace{\left[\frac{\partial y_2(x_1^*, x_2^*)}{\partial x_2} - \frac{\partial \tilde{y}_2(Y_1, x_2^*)}{\partial x_2} \right]}^{\text{NSE}} + \overbrace{\left[\frac{\partial \tilde{y}_2(Y_1, x_2^*)}{\partial x_2} - E_{\tilde{z}} \frac{\partial \tilde{y}_2(Y_1 - z, x_2^*)}{\partial x_2} \right]}^{\text{SSE}}.$$

If the TSE is positive, then player 2 chooses a larger input in response to an input-setting opponent than in response to an output-setting opponent (i.e., the direction of the strategic effect is the same as in the non-stochastic setting). If TSE is negative, the opposite is true.

The first component of the TSE, the NSE, is the effect identified in the non-stochastic model. Hence, as in our earlier analysis, NSE is positive and leads player 2 to choose a smaller input when facing an output-leading opponent than when facing an input-leading opponent. Thus, the basic strategic effect of the non-stochastic model persists.

Uncertainty introduces an additional strategic effect that is due not to the rival player's choice of strategic variable, but to the player's uncertainty about which particular strategy the rival player has chosen. The SSE captures the fact that when player 1's production is random, player 2 faces uncertainty about the marginal product of his input expenditure, and this uncertainty affects his optimal strategy. In response, player 2 will adjust his input usage in order to set its expected marginal product equal to its marginal cost. Whether this adjustment leads player 2 to increase or decrease his input usage depends on whether $\frac{\partial \tilde{y}_2}{\partial x_2}$ is concave or convex in y_1. Although natural assumptions on the production function do not determine the sign of the SSE, it is easily computed for a particular input-output relationship.

If uncertainty tends to increase x_2 (i.e., the SSE is positive), then this effect reinforces the NSE, and player 2's overall reaction is larger against an input-leading opponent than against an output-leading opponent (i.e., TSE is positive). If the SSE is negative, then this effect works against the NSE. If the SSE is negative and sufficiently strong, it may even reverse the overall impact of adopting an output-setting strategy; the TSE may be negative and player 2's reaction against an output-setting opponent may be larger than his reaction against an input setting opponent. Of course, for this to happen $\frac{\partial \tilde{y}_2}{\partial x_2}$ must be convex in y_1 and uncertainty must be sufficiently important. When uncertainty is small to moderate, this effect is unlikely to swamp the strategic effect.

The overall decision of whether to lead input or output must consider both the direct and strategic effects. When the TSE is positive, then the strategic effect favors input setting if $\frac{\partial y_1}{\partial x_2} > 0$ and output setting if $\frac{\partial y_1}{\partial x_2} < 0$. Conversely, when the TSE is negative, then the strategic effect favors output setting if $\frac{\partial y_1}{\partial x_2} > 0$ and input setting if $\frac{\partial y_1}{\partial x_2} < 0$. However, the direct effect (OUE) always favors input-setting.

Table 2 summarizes player 1's preferred leading variable when facing an input-leading opponent.

	TSE Positive $\frac{\partial y_1}{\partial x_2} > 0$	TSE Positive $\frac{\partial y_1}{\partial x_2} < 0$	TSE Negative $\frac{\partial y_1}{\partial x_2} > 0$	TSE Negative $\frac{\partial y_1}{\partial x_2} < 0$
Weak OUE	Input	Output	Output	Input
Strong OUE	Input	Input	Input	Input

Table 2: Player1's optimal choice of leading variable when player 2 leads input.

When the impact of uncertainty is relatively mild, the operative case is where the OUE is weak and the TSE is positive, which establishes the following corollary.

Corollary 1 . Provided the impact of uncertainty is not too large, against an input-leading opponent it is a dominant strategy in this game for player 1 to set input if $\frac{\partial y_1}{\partial x_2} > 0$ and set output if $\frac{\partial y_1}{\partial x_2} < 0$.

Corollary 1 establishes that the basic results of the non-stochastic analysis are robust to the introduction of a moderate amount of uncertainty. When both players' outputs experience stochastic shocks (i.e., realized output can be written as $Y_i(x_i, x_j, z_i) = y_i(x_i, x_j) + z_i$, where z_i is the realization of a firm-specific shock), the analysis becomes even more complicated. While there is still a direct advantage to setting input when the rival player is an input-setter, when the rival player is an output-setter the direction of the direct effect cannot be signed. In particular, the nature of these effects may depend on the distributions of the shocks and their correlation, among other things. Nevertheless, because the non-stochastic strategic effect persists and the effects of uncertainty are small for mild uncertainty, the basic results on the optimality of input-setting or output-setting continue to hold, provided the uncertainty is not too large.[27]

A stochastic contest

Although it is difficult to make general statements regarding the players' decisions whether to lead input or output in general stochastic environments, much can be said in particular environments. To investigate the players' simultaneous incentives when both players' outputs are subject to stochastic shocks, we now turn our attention to a stochastic version of the contest discussed in Example 1. Let $\tilde{z}_1$ denote a zero-mean random shock to player i's winning probability with typical realization z_1. Let $\tilde{z}_2 = -\tilde{z}_1$. The players' output functions are:

$$y_i(x_i, x_j, z_i) = A\left(\frac{x_i}{x_i + x_j + \delta} + z_i\right).$$

The stochastic shocks to the players' winning probabilities are perfectly negatively correlated. Thus, if player 1 has been lucky, player 2 has necessarily been unlucky (and vice versa).

[27] In this section, we have considered one particular specification of uncertainty. However, there could be others. Depending on the nature of the shock (i.e., additive, multiplicative, etc.) the direction of the effects identified in this section may change. Nevertheless, the conclusion that the results on the dominance of input- or output-setting when uncertainty is mild will remain valid.

Again, this game can be thought of as the last stage of a multi-period competition. The realization of the shock ($\tilde{z}_i$) captures the state of the game at the start of the final period, which depends on factors that are unknown to the players at the time they must choose their strategies and is, therefore, stochastic. Alternatively, we can interpret the game as the players' input choices representing flows of expenditure that take place over time, during which random events may either increase or decrease the likelihood of winning the contest. In this interpretation, we do not allow for the players to vary their input flows over time.

To begin, we define the following stochastic equivalents to functions $x_i(y_i, y_j)$, $\tilde{y}_i(x_i, y_j)$, and $\tilde{x}_i(y_i, x_j)$:

$$
\begin{aligned}
x_i(y_i, y_j, z_i) &= \frac{(y_i - Az_i)\,\delta}{A - y_1 - y_2} \\
\tilde{y}_i(x_i, y_j, z_i) &= \frac{(A + y_j)\,x_i + Az_i\delta}{x_i + \delta} \\
\tilde{x}_i(y_i, x_j, z_i) &= \frac{y_i(x_j + \delta) - Az_i(x_j + \delta)}{A + Az_i - y_i}
\end{aligned}
$$

Note that $y_i(x_i, x_j, z_i)$, $x_i(y_i, y_j, z_i)$, and $\tilde{y}_i(x_i, y_j, z_i)$ are linear in z_i. Therefore, $E_{\tilde{z}_i} y_i(x_i, x_j, z_i)$, $E_{\tilde{z}_i} \tilde{y}_i(x_i, y_j, z_i)$ and $E_{\tilde{z}_i} x_i(y_i, y_j, z_i)$ are independent of the distribution of $\tilde{z}_i$. Similarly, $E_{\tilde{z}_i} \frac{\partial y_i}{\partial x_i}$, $E_{\tilde{z}_i} \frac{\partial x_i}{\partial y_i}$, and $E_{\tilde{z}_i} \frac{\partial \tilde{y}_i}{\partial x_i}$ are also independent of the distribution of $\tilde{z}_i$. The remaining function, $\tilde{x}_i(y_i, x_j, z_i)$, is convex in z_i, as is its derivative $\frac{\partial \tilde{x}_i}{\partial y_i}$.

These observations imply the following proposition.

Proposition 8 . In the stochastic contest, if player j leads input x_j, player i strictly prefers to lead input. If player j leads output y_j, player i is indifferent between leading input and leading output.

Proposition 8 establishes that Proposition 1 no longer holds in the stochastic setting; players are not indifferent between input leading and output leading. Interestingly, the perfect negative correlation between the two players' shocks is responsible for the lack of a direct advantage to one or the other strategy when setting output. Written as a function of both shocks, $x_i(y_i, y_j, z_1, z_2) = (y_i - Az_i)\,\delta / (A - y_1 - z_1 - y_2 - z_2)$. Since $z_1 = -z_2$, the denominator is non-stochastic, leaving the resulting expression linear in the uncertainty.

Proposition 9 characterizes how the strategic effects change when uncertainty is incorporated into the model.

Proposition 9 . In the stochastic contest, if player i chooses:

(a) input against an input-leading opponent, uncertainty does not affect i's best response to x_j.

(b) input against an output-leading opponent, uncertainty does not affect i's best response to y_j.

(c) output against an input-leading opponent, uncertainty reduces i's best response to any x_j.

(d) output against an output-leading opponent, uncertainty does not affect i's best response to y_j.

Proposition 9 implies that the nature of the total strategic effect does not change when a player leads input. An input-leader chooses a larger input when facing an input-leading rival than when facing an output-leading rival. Part (c) of the proposition suggests that the analysis is more complicated if the player leads output. The effect identified in part (c) tends to mitigate the strategic effect. It reduces the extent to which an output leader chooses a smaller output when facing another output leader than when facing an input leader. If this effect is sufficiently large, it could reverse the overall direction of the strategic effect, and thus an output leader might be more aggressive when facing an output leader than when facing an input leader. However, once again this is only possible if there is a relatively high degree of uncertainty.

Corollary 2 . Provided the effects of uncertainty are not too large, in the stochastic contest game each player has a dominant strategy to lead output.

When the effects of uncertainty are large enough, it may be that output-setting is no longer a dominant strategy in the meta-game. In the stochastic contest uncertainty increases the relative attractiveness of leading input because of (i) the direct advantage identified in Proposition 8, and (ii) part (c) of Proposition 9, according to which increased uncertainty decreases the strategic advantage to output-leading when facing an output-leading opponent. To illustrate, in a non-stochastic R&D contest it is optimal for the firms to lead output. However, with enough uncertainty about the feasibility of the innovation, specifying an output target (e.g., innovation date or probability of being first to market) forces the firm to bear all of the risk of the difficulty of the innovation on the input-side; it commits to spend whatever is necessary to innovate by the target date. If the cost of this risk is sufficiently high, the firm will forego the strategic benefits of committing to an output strategy in order to avoid the risky cost of meeting the output target.

6 Applications

We now turn to illustrating how this analysis can inform our understanding of real-world strategic situations. We begin by returning to the example that started the paper, the decision to go to the moon. We then provide a number of other examples where the choice of leading variables plays an important role. We do not pretend to entirely explain these interactions. Rather, our goal is to illustrate that the strategic effects we identify are real, common, and potentially important.

The space race revisited

Consider once again the hypothetical question of whether President Kennedy was right to adopt an output strategy, or whether an input strategy might have been better. According to our theory, the key to answering this question is whether increasing spending on the U.S. space program would have increased or decreased the likely success of the Soviet program.

There is significant evidence that the benefit derived from winning the space race consisted primarily of the prestige associated with being the first nation to the moon. According to the report of Kennedy's advisory committee on the exploration of space, "during the next few years, the prestige of the United States will in part be determined by the leadership we demonstrate in space activities."[28] The scientific benefits associated with being first to the moon, as opposed to being second to the moon or achieving advances in earth-orbit programs, were considered minimal, and focusing on the moon race may have even taken resources away from military rocket programs.[29] Hence the game between the U.S. and U.S.S.R. was one of strictly opposed outputs. Seen in this light, announcing an output target was, indeed, prudent. By declaring to the Soviets and the world that the U.S. would be the first to the moon, no matter what the cost, Kennedy signaled to the Soviets that increasing spending in an effort to prevent the U.S. from getting to the moon first would ultimately be fruitless.

As a final note on the space race, the strategic effects we have identified are only relevant if the first mover's announcement is seen by the other player as a credible commitment, and establishing credibility in an environment as filled with technological and political uncertainty as this one is

[28] Quoted in Beschloss, (1997), p. 54.

[29] Testifying before Congress in 1958, NASA deputy administrator Hugh Dryden described the Defense Department's manned spaceflight proposal as having "about the same technical value as the circus stunt of shooting the young lady from the gun." (quoted in Beschloss, 1997, p. 36).

an incredibly challenging task. Still, Kennedy's rhetorical power may have served him well here. Indeed, Kennedy's commitment was strong enough that putting a man on the moon by the end of the decade persisted as a national goal even after his death. If this goal had been announced by a president less able to rally the country behind him, the declaration may have been dismissed as mere cheap talk.

Research and development

Consider two firms that simultaneously undertake research and development programs aimed at bringing similar products to market. In such games, there are a number of possible effects of the intensity of the rival firm's program. Since the firms are working on related products, there may be positive innovative spillovers, in which case $\frac{\partial y_i}{\partial x_j} > 0$. Each firm has an incentive to free-ride off the efforts of the other. In this case, Proposition 4 shows that the dominant meta-game strategy is to set input, since by doing so the firm can signal to its rival that it will not reduce its input in response to the rival's expenditures. On the other hand, since both firms are developing similar products, they may be competing for scarce resources such as raw materials, scientific talent, or marketing channels. In this case, an increase in intensity by the other firm is harmful, $\frac{\partial y_i}{\partial x_j} < 0$, and Proposition 4 implies that the optimal meta-game strategy is to set output. Finally, if the products are patentable, being the first to market might present the ultimate reward, in which case the game is of opposing interests, and once again the optimal strategy is to set output.

Negotiations

Consider two parties who meet to negotiate the division of an asset such as land, the gains from a trade relationship, or the terms of a cease-fire. Here, the input is the time and effort the party puts into the negotiation, and the output is the share of the asset that the player wins. Clearly, this is a game of strictly opposed outputs, and thus, leading output is a dominant strategy.

The strategic difference between leading input and output in this type of situation plays a prominent role in one of the classic motivational stories in negotiation strategy. In his book *You Can Negotiate Anything* (Cohen 1980), Herb Cohen tells of when he was sent to Japan on his first major negotiation. When Cohen arrived in Japan, his hosts asked him when his return flight was, ostensibly in order to reserve the company's limousine for him. Cohen told them his itinerary, at which point his hosts began to wine and dine him, refusing to begin serious negotiations until just before Cohen's deadline. The deal was finalized on the way to the airport, and Cohen, desperate not to return home empty-handed, was forced to make many unnecessary concessions.

What went wrong for Cohen? By telling his opponents his deadline, he implicitly adopted an input-leading strategy. Knowing this, his opponents only had to be more patient (i.e., use more inputs) than Cohen to gain the advantage. The analysis in this paper suggests that Cohen's response to the question about his return flight should have been to say that he was prepared to stay as long as necessary to achieve his goals, eliminating the other side's ability to exploit his deadline. In other words, he should have adopted an output strategy.

Corporate strategy

Suppose two auto manufacturers are making decisions about capacity and sales for the next decade. All else equal, the larger the total capacity in the industry, the lower are prices. Hence profit from sales is decreasing in the other firm's capacity (i.e., $\frac{\partial y_i}{\partial x_j} < 0$). In this case, it is optimal for the players to adopt an output-leading posture. By specifying sales targets and committing to build or destroy capacity to meet them, each firm signals to its rival that it will not be drawn into building more capacity in order to increase unit sales while driving prices down to the point where both firms are worse off. Interestingly, in this context output-setting may be a provide a mechanism for tacit collusion.

On the other hand, consider firms that produce complementary products, such as computer hardware and software. For hardware, let the input variable be capacity. For software, let the input variable be dollars spent on developing new programs. In either case, the output is profit before spending on capacity or development. In this case $\frac{\partial y_i}{\partial x_j} > 0$: greater availability of hardware makes software more profitable and vice versa. The theory implies that players should lead inputs. By committing to provide a certain number of computers to the market, software manufacturers are convinced that the hardware manufacturer will not try to restrict the number of machines available, using the value created by the software to inflate hardware prices. Similarly, by committing to spend a certain amount on software development, the software company commits to a strategy of developing more and better products rather than selling the same old product and relying on the large number of computers to generate sales volume.

Advertising

In the practice of marketing, the choice between leading variables plays an important role, and both types of leading variables are observed. While some firms set advertising budgets at the start of the year, other companies, such as Anheuser-Busch and Unilever, avoid a commitment to a budget in favor of a detailed specification of measurable output goals such as reach, frequency, production costs and even desired sales for the campaign.[30]

The results presented here state that if advertising by one firm increases the sales of the other firm, the firms should lead with inputs because by doing so they are able to commit not to free ride off their rival's spending. On the other hand, if increasing advertising decreases the sales of the other firm, then the firms should choose output strategies since by doing so they commit to not enter into an advertising war.

Advertising raises the interesting possibility of dissimilar players. For example, it might be that advertising by a popular hotel chain promoting a specific location increases the demand for all hotels in the area, while advertising by one of the local hotels in the area primarily steals business away from the chain. In this case, the theory suggests that the chain should lead input and the local hotel should lead output. By leading output, the local hotel signals to the chain that it will not attempt to steal too much of the chain's business, and by leading input, the chain signals that it will not free ride off of the local hotel's generosity.

7 Conclusion

We have considered competitive environments from a slightly unusual perspective. Rather than taking the game as fixed and studying the equilibrium, we have instead focused on how, through the choice of a leading variable, a player can affect the game that is being played, and consequently influence the ultimate equilibrium outcome. In the course of our analysis, we have identified several general principles. First, the importance of a player's choice of leading variable is not that it affects the possibilities open to him, but rather that it influences the other player's behavior. Second, as long as the sign of $\frac{\partial y_i}{\partial x_j}$ does not depend on the input vector at which it is evaluated, either leading input or leading output is a dominant strategy in the meta-game. Third, as a consequence of these

[30]For a complete discussion of advertising budgets see Kotler (2000). Miller and Pazgal (2005) considers the question of input setting vs. output setting in a marketing/advertising context.

dominant strategies, the meta-game exhibits a unique equilibrium.

Our goal has been to produce a theory that can be helpful in understanding real-world strategic interactions, and the extensions to the basic theory and applications are meant to give a flavor of how this might work. In some of the applications, such as the space race and Herbert Cohen's negotiation story, we have attempted to apply the model to highly complex, unstructured interactions. While there is certainly more to these situations than a simple choice between input and output strategies, we believe that considering the strategic situation in terms of this choice helps to explain why things happened the way they did, as well as how the players may have improved their outcomes by acting differently. In short, while we admit that we have not told the *whole* story, we believe that, in a wide variety of circumstances, the issues explored in this paper may account for an important *part* of the story.

Appendix A

Properties of the input-output relationship

The following three observations are used in the proofs.

Observation 1: Differentiating $y_i\left(\tilde{x}_i\left(y_i, x_j\right), x_j\right) \equiv y_i$ with respect to y_i yields $\frac{\partial y_i}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial y_i} = 1$, or

$$\frac{\partial \tilde{x}_i}{\partial y_i} = \frac{1}{\frac{\partial y_i}{\partial x_i}}. \tag{A1}$$

Differentiating a second time with respect to y_i verifies that $\tilde{x}_i$ is convex in y_i.

Observation 2: Differentiating $y_1\left(x_1\left(y_1, y_2\right), x_2\left(y_2, y_1\right)\right) \equiv y_1$, and $y_2\left(x_2\left(y_2, y_1\right), x_1\left(y_1, y_2\right)\right) \equiv y_2$ with respect to y_1 and y_2 yields four equations in four unknowns, allowing us to solve for the partial derivatives of inputs with respect to outputs (i.e., $\frac{\partial x_1}{\partial y_1}, \frac{\partial x_1}{\partial y_2}$, etc.) in terms the partial derivatives of outputs with respect to inputs (i.e., $\frac{\partial y_1}{\partial x_1}, \frac{\partial y_2}{\partial x_2}$, etc.). In particular, this implies that

$$\frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}}. \tag{A2}$$

Observation 3: Differentiating identity $x_i\left(\tilde{y}_i\left(x_i, y_j\right), y_j\right) \equiv x_i$ with respect to x_i yields $\partial \tilde{y}_i/\partial x_i = 1/\left(\partial x_i/\partial y_i\right)$. Combining this with equation (A2) above,

$$\frac{\partial \tilde{y}_i}{\partial x_i} = \frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}}. \tag{A3}$$

Thus when players are similar, $\frac{\partial \tilde{y}_i}{\partial x_i} < \frac{\partial y_i}{\partial x_i}$. The opposite inequality holds for dissimilar players.

Proofs

Proof of Proposition 1. Suppose player j leads input but player i leads output. Player i chooses y_i to maximize $y_i - \tilde{x}_i\left(y_i, x_j\right)$ subject to admissibility of $\left(y_i, x_j\right)$. Differentiating with respect to y_i and setting the result equal to zero yields (for an interior solution):

$$\frac{\partial \tilde{x}_i\left(y_i^x\left(x_j\right), x_j\right)}{\partial y_i} \equiv 1, \tag{A4}$$

where function $y_i^x\left(x_j\right)$ is player i's optimal output-reaction to player j's choice of x_j. Combining (A1) and (A4) implies:

$$\frac{\partial y_i\left(\tilde{x}_i\left(y_i^x\left(x_j\right), x_j\right), x_j\right)}{\partial x_i} \equiv 1, \tag{A5}$$

and hence $\tilde{x}_i\left(y_i^x\left(x_j\right), x_j\right) \equiv R_i\left(x_j\right)$, by uniqueness of $R_i\left(x_j\right)$. That is, holding fixed x_j, $y_i^x\left(x_j\right)$ implies the same relationship between x_i and x_j as $R_i\left(x_j\right)$. Similar arguments for other combinations of strategic variables complete the proof. *Q.E.D.*

Proof of Proposition 2. By (A2) and (A3),

$$\frac{\partial \tilde{y}_i}{\partial x_i} = \frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}} < \frac{\partial y_i}{\partial x_i} \tag{A6}$$

when the players are similar. When the players are dissimilar, the opposite inequality holds. Define $\phi\left(x_i, x_j\right)$ to be player i's marginal utility of an increase in x_i as a function of both players' input choice: $\phi_i\left(x_i, x_j\right) \equiv \frac{\partial y_i(x_i,x_j)}{\partial x_i} - 1$. Equation (2) implies that $\phi_i\left(x_i, x_j\right) = 0$ whenever $x_i = R_i\left(x_j\right)$. Since $\phi_i\left(x_i, x_j\right)$ is just the partial derivative of $y_i\left(x_i, x_j\right) - x_i$ with respect to x_i and $y_i\left(x_i, x_j\right)$ is concave in x_i, for similar players $\phi_i\left(x_i, x_j\right) > 0$ below $R_i\left(x_j\right)$ (that is, nearer to the $x_i = 0$ axis), and $\phi_i\left(x_i, x_j\right) < 0$ above $R_i\left(x_j\right)$. See Figure 1, Panel A.

Fix an input vector $\left(\hat{x}_i, \hat{x}_j\right)$ such that $\hat{x}_i = r_i\left(\hat{x}_j\right)$, and let $\hat{y}_j = y_j\left(\hat{x}_i, \hat{x}_j\right)$. By (A6), when players are similar and $\frac{\partial \tilde{y}_i\left(x_i^y(\hat{y}_j),\hat{y}_j\right)}{\partial x_i} = 1$, $\frac{\partial y_i(r_i(x_j),x_j)}{\partial x_i} > 1$. Therefore $\phi_i\left(x_i, x_j\right) > 0$ whenever $x_i = r_i\left(x_j\right)$. When players are dissimilar, the opposite inequality holds in (A6), and the opposite argument follows. See Figure 1, Panel B. *Q.E.D.*

Proof of Lemma 1. To begin, note that the slope of firm i's profit isoquant through $\left(x_1, x_2\right)$ is given by $dx_j/dx_i = -\left(\left(\partial y_i/\partial x_i\right) - 1\right)/\left(\partial y_i/\partial x_j\right)$. That $\partial \pi_i\left(R_i\left(x_j\right), x_j\right)/\partial x_j$ has the same sign as $\partial y_i/\partial x_j$ follows from the definition of the reaction function (equation (2)) and the fact that player i's profit isoquants are vertical along $R_i\left(x_j\right)$ and increasing x_j affects player i's output but not his input. To see that $\partial \pi_i\left(r_i\left(x_j\right), x_j\right)/\partial x_j$ has the same sign, note that player i's profit along his reaction curve when player j leads with outputs must be monotone in the output-output space. By the invertibility assumptions, profit must then be monotone along the projection of this reaction curve into the input space, $r_i\left(x_j\right)$. To prove that profit must increase in the same direction as it does along $R_i\left(x_j\right)$, suppose that $\frac{\partial y_i}{\partial x_j} > 0$, and consider Figure 4.[31]

Point (a) is an arbitrary point along $R_i\left(x_j\right)$. The profit isoquant through point (a) is vertical by the definition of $R_i\left(x_j\right)$. Since $\frac{\partial y_1}{\partial x_2} > 0$, the upper level-set of the profit function lies to the right of point (a). Consequently, the point where this profit isoquant intersects $r_i\left(x_j\right)$, point (c), must involve a strictly larger value of x_j. On the other hand, moving down along the dotted line

[31] Figure 4 is drawn for similar players, but the same argument applies for dissimilar players.

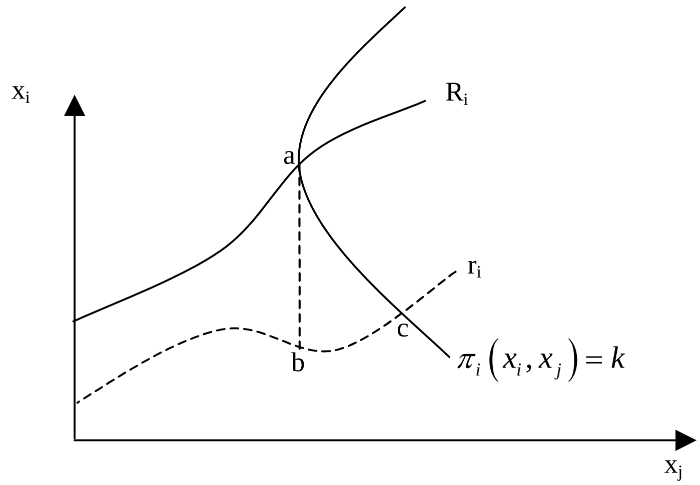

Figure 4: Profit is monotone along r_i.

from point (a) to point (b) profit must decrease. Thus profit is lower at (b) than (c). Since the original selection of point (a) was arbitrary, it must increase whenever x_j increases along $r_i(x_j)$. Similar arguments apply when $\frac{\partial y_i}{\partial x_j} < 0$ or players are dissimilar. *Q.E.D.*

Proof of Proposition 3. By Lemma 1, if $\frac{\partial y_1}{\partial x_2} > 0$ then $\pi_1^{xx} > \pi_1^{yx}$ and $\pi_1^{xy} > \pi_1^{yy}$, and therefore input-setting is dominant for player 1. The remaining claims follow from Lemma 1 in a similar manner. *Q.E.D.*

Proof of Proposition 4. Follows immediately from Proposition 3. *Q.E.D.*

Proof of Proposition 5. Figure 5 illustrates the proof.

Suppose $\frac{\partial y_i}{\partial x_j} > 0$. Since point yy lies on r_2 and r_1, and these lie nearer the origin than R_2 and R_1, respectively, yy must involve smaller input usage than xx, which lies on R_2 and R_1. Since $\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0$ and both x_1 and x_2 are smaller at yy than xx, outputs must also be smaller at yy than xx. Since $\frac{\partial y_2}{\partial x_1} > 0$, moving horizontally along the dotted arrow between yy and R_2 increases player 2's payoff, and, for the same reason, moving up along R_2 also increases 2's payoff. Hence 2's payoff must be larger at xx than at yy. Player 2's payoff must be larger at xx than yx since moving right along the dotted arrow to R_2 increases payoff, and then moving up R_2 to xx also increases payoff. The same argument applies for player 1 with the roles reversed.

When $\frac{\partial y_i}{\partial x_j} < 0$, the argument that inputs must be larger at xx than yy still applies. Outputs cannot be ranked. As for payoffs, moving down along R_2 from xx to the point where the lower

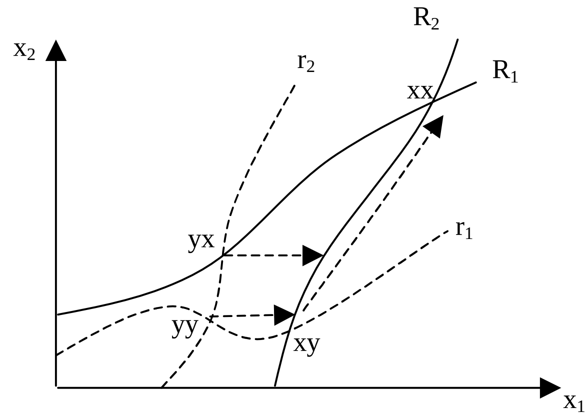

Figure 5: Payoff comparisons: strategic complements.

dotted arrow intersects it decreases 2's payoff, as does moving horizontally along the dotted arrow to yy. Thus 2 must earn higher payoffs at yy than xx. Since yx lies up r_2 from yy, 2 must earn higher payoff at yy than yx. However, if xy lies below the lower horizontal dotted arrow, as it does in the diagram, then it may be that 2's payoff at xy is higher than at yy. The symmetric argument applies for player 1. *Q.E.D.*

Proof of Proposition 6. Suppose $\frac{\partial y_1}{\partial x_2} > 0$, and consider the case where player 1 chooses output strategy y_1^*. In response to this, player 2 chooses x_2 in order to maximize $\tilde{y}_2(x_2, y_1^*) - x_2$. Let x_2^* be the solution to player 2's problem, and let $x_1^* = \tilde{x}_1(y_1^*, x_2^*)$ be the implied input usage by player 1. Now, suppose that instead of player 1 choosing y_1^*, player 1 had instead chosen x_1^*. By Proposition 2, in response to x_1^*, player 2 chooses $x_2' > x_2^*$. Since $y_1(x_1^*, \tilde{x}_2) > y_1^*$, player 1 earns higher payoff when choosing x_1^*.[32] Hence y_1^* could not have been optimal. The proof is illustrated in Figure 6, Panel A.

The proof of the second part is similar. Suppose $\frac{\partial y_1}{\partial x_2} < 0$, and that player 1 chooses input strategy x_1^*. In response to this, player 2 chooses x_2 in order to maximize $y_2(x_1^*, x_2) - x_2$. Let x_2^* be the solution to player 2's problem, and let $y_1^* = y_1(x_1^*, x_2^*)$. Suppose that player 1 had instead chosen y_1^*. Since $\frac{\partial y_1}{\partial x_2} < 0$, the set of points where $y_1(x_1, x_2) = y_1^*$ slopes upward. Hence Player 2's best response to y_1^* is $x_2^{'} < x_2^*$. And, since $\frac{\partial y_1}{\partial x_2} < 0$, player 1 the input, $x_1^{'}$, needed to sustain y_1^*

[32] The slope of an output isoquant for player 1 is $-(\partial y_1/\partial x_1)/(\partial y_1/\partial x_2)$, whose sign is opposite that of $\frac{\partial y_1}{\partial x_2}$.

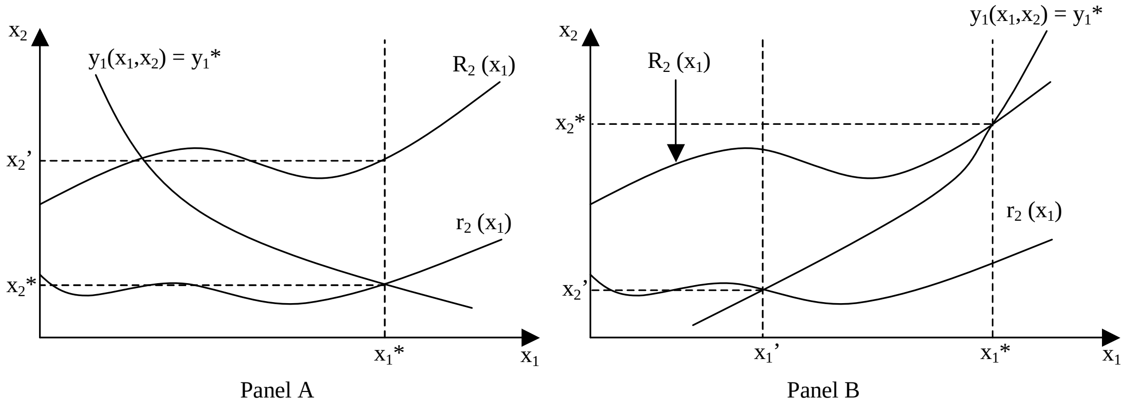

Figure 6: If $\frac{\partial y_1}{\partial x_2} > 0$, output strategies are dominated. If $\frac{\partial y_1}{\partial x_2} < 0$, input strategies are dominated.

in response to x_2' is smaller than x_1^*. Hence player 1 earns higher payoff, and choosing x_1^* cannot be optimal. The proof is illustrated in Figure 6, Panel B. *Q.E.D.*

Proof of Proposition 7, Fix player 2's input choice at x_2^*, and suppose that player 1 leads output. Let Y_1^* be the profit-maximizing output target given x_2^*. Expected input usage is given by $E_{\tilde{z}}\tilde{x}_1\left(Y_1^* - z, x_2^*\right)$. By convexity of $\tilde{x}_1\left(y_1, x_2\right)$ in y_1, $E_{\tilde{z}}\tilde{x}_1\left(Y_1^* - z, x_2^*\right) > \tilde{x}_1\left(Y_1^*, x_2^*\right)$. That is, the expected cost of achieving expected output Y_1^* under input setting is greater than the cost of producing Y_1^* if there were no uncertainty. Let $x_1^* = \tilde{x}_1\left(Y_1^*, x_2^*\right)$ denote this cost.

To show that input-setting dominates output-setting, suppose that player 1 leads input, and let $\hat{x}_1$ maximize $y_1\left(x_1, x_2^*\right) + \tilde{z} - x_1$. Optimal profit is given by:

$$y_1\left(\hat{x}_1, x_2^*\right) - \hat{x}_1 \geq y_1\left(x_1^*, x_2^*\right) - x_1^* > Y_1^* - E_{\tilde{z}}\tilde{x}_1\left(Y_1^* - z, x_2^*\right).$$

The first inequality follows from optimality of $\hat{x}_1$, while the second follows from convexity of $\tilde{x}_1\left(y_1, x_2\right)$ in y_1. Finally, note that $Y_1^* - E_{\tilde{z}}\tilde{x}_1\left(Y_1^* - z, x_2^*\right)$ is the maximal expected profit of an output setter. *Q.E.D.*

Proof of Proposition 8. Let $i = 1$ and $j = 2$. Suppose player 2 sets input, and let player 1's best output choice be y_1^*. Player 1's expected profit is

$$y_1^* - E_{\tilde{z}_1}\tilde{x}_1\left(y_1^*, \bar{x}_2, z_1\right).$$

Let x_1^* be such that $y_1^* = E_{\tilde{z}_1}y_1\left(x_1^*, x_2, \tilde{z}_1\right)$. If player 1 sets input and chooses x_1^*, expected profit is:

$$E_{\tilde{z}_1} y_1 \left(x_1^*, \bar{x}_2, z_1\right) - x_1^* = y_1 \left(x_1^*, \bar{x}_2, 0\right) - x_1^* = y_1^* - \tilde{x}_1 \left(y_1^*, \bar{x}_2, 0\right) < y_1^* - E_{\tilde{z}_1} \tilde{x}_1 \left(y_1^*, \bar{x}_2, z_1\right),$$

where the first and second equalities follow from the fact that $y_1 \left(x_1^*, \bar{x}_2, z_1\right)$ is linear in z_1 and $E\tilde{z}_1 = 0$ and the definition of y_1^*, and the inequality follows form concavity of $\tilde{x}_1 \left(y_1^*, \bar{x}_2, z_1\right)$ in z_1.

For the second part, suppose player 2 sets output, and let player 1's best output choice be y_1^*. Let $x_1^* = E_{\tilde{z}_1} \left(x_1 \left(y_1^*, \bar{y}_2, z_1\right)\right)$, and note that x_1^* does not depend on the distribution of $\tilde{z}_1$. Player 1's expected profit is:

$$y_1^* - E_{\tilde{z}_1} \left(x_1 \left(y_1^*, \bar{y}_2, z_1\right)\right) = y_1^* - x_1 \left(y_1^*, \bar{y}_2, 0\right) = \tilde{y}_1 \left(x_1^*, \bar{y}_2, 0\right) - x_1^* = E_{\tilde{z}_1} \tilde{y}_1 \left(x_1^*, \bar{y}_2, z_1\right) - x_1^*.$$

Thus, player 1 can achieve his maximal output-setting profit by setting input. Since the steps reverse, player 1 can also achieve his maximal input-setting profit by setting output, which completes the proof. *Q.E.D.*

Proof of Proposition 9. Suppose that player 1 leads input, and consider his reaction to an input-setting and output-setting opponent. If player 2 sets input, player 1 chooses x_1 to maximize $E_{\tilde{z}_1} \left(y_1 \left(x_1, x_2, z_1\right)\right) - x_1$, subject to admissibility, which for an interior solution yields first-order condition $E_{\tilde{z}_1} \frac{\partial y_1(x_1, x_2, z_1)}{\partial x_1} = 1$, or $\frac{\partial y_1(x_1, x_2, 0)}{\partial x_1} = 1$. Thus, because output is linear in z_1, uncertainty does not affect player 1's best response condition. Similar computations when player 1 sets input and player 2 sets output and when player 1 sets output and player 2 sets output show that uncertainty also does not affect player 1's best response condition in these cases. When player 1 sets output and player 2 sets input, player 1 chooses y_1 to maximize $y_1 - E_{\tilde{z}_1} \tilde{x}_1 \left(y_1, x_2, z_1\right)$, subject to admissibility, which for an interior solution has first-order condition $E_{\tilde{z}_1} \frac{\partial \tilde{x}(y_1, x_2, z_1)}{\partial y_1} = 1$. Since $\tilde{x} \left(y_1, x_2, z_1\right)$ is convex in z_1, $E_{\tilde{z}_1} \frac{\partial \tilde{x}(y_1, x_2, z_1)}{\partial y_1} > \frac{\partial \tilde{x}_1(y_1, x_2, 0)}{\partial y_1}$. *Q.E.D.*

Appendix B

In this Appendix, we show how the results extend to the case of general quasiconcave utility functions.

The basic analysis considers players whose preferences over input-output pairs are given by $y_i - x_i$. We have chosen this representation and maintain it for most of the paper because we believe it is the most natural one in many of the applications we consider. However, our results can be easily extended to the case where utility is a more general function of input and output, $u_i(y_i, x_i)$. Thus, in particular, it need not be the case that x_i and y_i are both stated in dollar terms. In this section, we briefly sketch the argument, and point to how the basic proofs of our original arguments remain valid.

Suppose that player i has utility function $u_i(y_i, x_i)$, which is increasing in y_i and decreasing in x_i. If player i chooses x_i in response to an input-setting opponent, he chooses x_i to maximize $u_i(y_i(x_i, x_j), x_i)$, subject to admissibility of (x_i, x_j), which for an interior solution has first-order condition:

$$\frac{\partial u_i}{\partial y_i}\frac{\partial y_i}{\partial x_i} + \frac{\partial u_i}{\partial x_i} = 0. \tag{B1}$$

If $u_i(y_i, x_i)$ is strictly quasiconcave, strict concavity of $y_i(x_i, x_j)$ in x_i implies (B1) is satisfied by a unique x_i for each x_j, which we once again denote $R_i(x_j)$.

If, on the other hand, player 1 leads output, he chooses y_i to maximize $u_i(y_i, \tilde{x}_i(y_i, x_j))$,subject to admissibility, which for an interior solution has first-order condition $\frac{\partial u_i}{\partial y_i} + \frac{\partial u_i}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial y_i} = 0$. Once again, the fact that $\frac{\partial y_i}{\partial x_i} = 1/\frac{\partial \tilde{x}_i}{\partial y_i}$ implies that player i's best response does not depend on his own strategic variable. Hence Proposition 1 extends to this case of general preferences.

Next, we show that Proposition 2 extends as well. Consider player i's best response to an output-leading opponent. He chooses x_i to maximize $u_i(\tilde{y}_i(x_i, y_j), x_i)$, subject to admissibility of (x_i, y_j), which for an interior solution has first-order condition:

$$\frac{\partial u_i}{\partial y_i}\frac{\partial \tilde{y}_i}{\partial x_i} + \frac{\partial u_i}{\partial x_i} = 0. \tag{B2}$$

Again, strict quasiconcavity of $u_i(y_i, x_i)$ along with convexity of $\tilde{y}_i(x_i, y_j)$ in x_i implies a unique solution to (B2). As before, when players are similar, $\frac{\partial \tilde{y}_i}{\partial x_i} < \frac{\partial y_i}{\partial x_i}$, and hence the same argument as was used in the proof of Proposition 2 shows that it extends to this environment as well. That is, for given x_j, the input player i chooses as a best response to x_j is larger when he believes his opponent to be an input-setter than when he believes is opponent is an output-setter (when players are similar). Propositions 3 and 4 follow from Propositions 1 and 2.

As a final point on extension, note that the results of the paper hold if either $u_i(y_i, x_i)$ is quasiconcave and $y_i(x_i, x_j)$ is strictly concave in x_i, or if $u_i(y_i, x_i)$ is strictly quasiconcave and $y_i(x_i, x_j)$ is concave. Hence we can also reproduce the Jéhiel and Walliser (1995) results, which consider strictly quasiconcave utility functions when the four control variables are related linearly.

References

[1] Beschloss, M. "Kennedy and the Decision to Go to the Moon." In R. Launius and McCurdy, H., eds., *Space Flight and the Myth of Presidential Leadership.* Urbana, IL: University of Illinois Press, 1997.

[2] Cheng, L. "Comparing Bertrand and Cournot Equilibria: a Geometric Approach." *RAND Journal of Economics*, Vol. 16 (1985a), pp. 146-152.

[3] Cheng, L. "Inverting Systems of Demand Functions." *Journal of Economic Theory*, Vol. 37 (1985b), pp. 202-210.

[4] Cohen, H. *You Can Negotiate Anything.* New York: Bantam Books, 1980.

[5] Dixit, A. and Nalebuff, B. "Making Strategies Credible." In Zeckhauser, R., ed., *Strategy and Choice.* Cambridge, MA: MIT Press, 1991.

[6] Jéhiel, P. and Walliser, B. "How to Select a Dual Nash Equilibrium." *Games and Economic Behavior*, Vol. 10 (1995), pp. 333-354.

[7] Klemperer, P., and Meyer, M. "Price Competition vs. Quantity Competition: The Role of Uncertainty." *RAND Journal of Economics,* Vol. 17 (1986), pp. 618-638.

[8] Kotler, P. *Marketing Management, Tenth Edition.* Upper Saddle River, NJ: Prentice Hall, 2000.

[9] Miller, N., and Pazgal, A. "The Equivalence of Price and Quantity Competition with Delegation," *Rand Journal of Economics*, Vol 32 (2001), pp. 284-301.

[10] Miller, N., and Pazgal, A. "Advertising Budgets in Competitive Environments." mimeo, 2005.

[11] Schelling, T.C. *The Strategy of Conflict*, Cambridge, MA: Harvard University Press, 1960.

[12] Singh, N. and Vives, X. "Price and Quantity Competition in a Differentiated Duopoly." *RAND Journal of Economics*, Vol. **15** (1984), pp. 546-554.

[13] Vives, X. *Oligopoly Pricing: Old Ideas and New Tools.* Cambridge, MA: MIT Press, 1999.