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Efficiency in Partnerships with Joint Monitoring

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This paper examines deterministic partnerships where a single partner observes the actions taken by a subset of the other partners and issues a report conditional on that observation. No other partner has any additional information. In such a model, whenever the observing partner can see the action chosen by at least one other partner, the efficient action vector can be sustained in a perfect Bayesian equilibrium by sharing rule that exhibits budget balance and limited liability. *Journal of Economic Literature* classification numbers: C7, D7, D8, L2.

1. INTRODUCTION

In the wake of Holmstrom [3], there have been a number of attempts to solve the problem of moral hazard in partnerships where output depends deterministically on the unobserved actions taken by the players. All of these have maintained the assumption that no partner has any information about the action taken by any other partner, and none has been able to implement the efficient action vector while maintaining limited liability and budget balance among the players. This paper considers the case where one player observes the actions taken by a subset of the other players and issues a report conditional on that observation. No other player has any information about the action taken by anyone else. It is shown that whenever the observing player can see at least one other player's action, efficiency, limited liability, and budget balance can be achieved simultaneously.

The model considered here is based on the partnership model used by Holmstrom [3], in which N players produce an output that depends

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deterministically on the unobservable actions taken by the players. Given a realized output, a sharing rule divides the output among the players. Holmstrom proves the general result that if budget balance is required the efficient output cannot be sustained in a Nash equilibrium. However, he also shows that if the budget balancing constraint is relaxed, efficiency can be sustained through the imposition of group penalties.

Legros and Matthews [5] show that in the Holmstrom model, budget balanced sharing rules can implement nearly efficient outcomes in a wide variety of games through the use of mixed strategies. However, in approximating the efficient output arbitrarily closely, the punishments used to enforce the equilibrium become arbitrarily large. Thus, as Legros and Matthews discuss, this equilibrium does not exhibit limited liability as the equilibrium expected output approaches the efficient output.

In this paper, Holmstrom's assumption that no player observes the action of any other player is replaced by the assumption that one player observes the actions taken by a subset of the remaining players and is able to issue a report conditional on that observation. The sharing rule divides the output based on the aggregate output and the report. It is shown that in this new game efficiency can be sustained in a perfect Bayesian equilibrium while maintaining budget balance and limited liability.

The assumption that one partner observes the actions taken by a subset of the other partners is not unreasonable. While in many everyday examples of partnerships it is clearly unrealistic to assume that a single principal observes the actions taken by all agents, in these same examples it is entirely natural that one agent should be able to observe the action taken by another agent. It seems inevitable, for example, that geographic neighbors (as in a cooperative farm) or neighbors in a production process (as in an assembly line) should observe each other's actions.

A related approach is taken by Ma [6], who shows that in a stochastic production model with a principal and many agents whose actions are mutually observable (although unobservable to the principal), there exists a mechanism that implements the efficient output as the unique subgame perfect equilibrium of the game. However, Ma is concerned with implementing the efficient action at the least cost to the principal, and consequently he does not consider budget balance among the agents or moral hazard on the part of all of the players, both of which are of central importance in the partnership literature. In addition, Ma's unique implementation comes at the expense of more complete information being available to some of the players.

Other approaches to the problem of moral hazard in partnerships have considered stochastic production functions as in Legros and Matsushima [4] and Williams and Radner [10], risk averse agents as in Rasmusen [9], and repeated games as in Radner [8] and Radner, Myerson and

Maskin [7]. Ben-Porath and Kahneman [1] consider joint monitoring as it relates to folk theorems in repeated games of imperfect information.

Section 2 of this paper presents the basic model of a deterministic partnership with joint monitoring. Section 3 contains the main results of the paper, characterizing the circumstances under which efficiency is and is not sustainable in this environment. Section 4 concludes.

2. MODEL AND ASSUMPTIONS

The model considered here is based on the partnership model used by Holmstrom [3]. A partnership consists of a set of players I, indexed by $i = 1, ..., N$. Each player i chooses an action a_i from a feasible set of actions A_i. The vector $(a_1, ..., a_N)$ is denoted by a. The output produced by the partnership is given by the non-negative function $y\colon \times_{i=1}^{N} A_i \to \Re$. Throughout the paper, arbitrary outputs will be denoted by y, and the output associated with action vector a will be denoted by $y(a)$.

Each player incurs a disutility of taking action a_i that is given by the nonnegative function $v_i(a_i)$. Utility is quasilinear in output and effort, taking the form $u(z, a_i) = z - v_i(a_i)$, where z is the share of output assigned to player i, and a_i is the action he took.

In order for the problem to have a well defined and interesting solution, make the following assumption:

$$
\text{(A0)} \begin{cases} \text{(i)} & \text{There exists a unique } a^* \text{ such that} \\ & a^* = \underset{a \in \times_{i=1}^{N} A_i}{\operatorname{argmax}}\, y(a) - \sum_{i=1}^{N} v_i(a_i) \\ \text{(ii)} & y(a^*) - \sum_{i=1}^{N} v_i(a_i^*) > 0 \\ \text{(iii)} & \text{If } \exists i \text{ such that } y(a_i, a_{-i}^*) = y, \text{ then } \forall j, \exists! a_j \\ & \text{such that } y(a_j, a_{-j}^*) = y. \end{cases}
$$

Part (iii) of A0 is made for notational simplicity and is without loss of generality. Throughout the paper, let $y^* \equiv y(a^*)$. Also, let $a_i(y) = \{a_i \mid a_i \in A_i \text{ and } y(a_i, a_{-i}^*) = y\}$, and note that (A0) implies that $a_i(y)$ is single valued.

Consider the case where player 1 perfectly observes the actions taken by a subset of the players. In order to formalize the information structure, partition the set of players I into the following three subsets. The first subset, $\{1\}$, consists of player 1 alone. The second subset, I_1, consists of the set of players beside player 1 whose actions are observed by player 1. Note that $1 \notin I_1$. The final subset, I_2, consists of the players whose actions are not observed by player 1. We generalize the notation above in the usual way, letting a_{I_1} refer to the action vector taken by I_1, A_{I_1} refer to the action set $\times_{i \in I_1} A_i$, etc.

The game then proceeds as follows:

Stage 1. All players choose actions knowing the sharing rule that will be implemented.

Stage 2. Player 1 sees the actions of the players in I_1 and issues a report conditional on the observation.

Stage 3. The sharing rule divides the output based on the realized output and the report.

Strategies for players other than player 1 involve only choosing an action. A strategy for player 1 involves both choosing an action and a *reporting rule*, $P: A_1 \times A_{I_1} \to R$, which specifies what player 1 will report at each of his information sets at stage 2. For the impossibility results in Propositions 1 and 2, the range of the reporting rule is left unspecified. In the case where $N \geqslant 3$ and I_1 and I_2 are both non-empty, a binary report taking values in the set $\{G, B\}$ is sufficient to sustain efficiency. When $N \geqslant 3$ and I_2 is empty but I_1 is not, taking the range of the reporting rule to be the set of all subsets of I_1 is sufficient to sustain efficiency.

The sharing rule z is a map from pairs (y, R) of realized output y and report R to $\mathfrak{R}^N_+$, with $z_i(y, R)$ being the share assigned to player i. There are several desirable characteristics of sharing rules for this game. First, the sharing rule z *maintains budget balance* if for all vectors of actions a and reports R, $\sum_{i=1}^N z_i(y, R) = y$. Second, the sharing rule *exhibits limited liability* if there exists a k such that for all y, R, and i, $z_i(y, R) \geqslant k$. Finally, a sharing rule z *sustains efficiency* if there exists a reporting rule P^* such that the strategy profile where every player other than player 1 plays his efficient action, a_i^*, and player 1 plays his efficient action a_1^* and the reporting rule P^*, forms a perfect Bayesian equilibrium of the game.

3. RESULTS

The results in this section characterize the circumstances under which efficiency can and cannot be sustained by a sharing rule that exhibits budget balance and limited liability. The first two propositions borrow heavily from Legros and Matthews [5]. Propositions 1 and 2 show that if player 1 cannot see the actions of any other player, or if $N = 2$, efficiency is sustainable with budget balance and limited liability if and only if the aggregate incentive to deviate is small. Proposition 3 shows that if player 1 can see the actions of some but not all of the other players (I_1 and I_2 are non-empty), efficiency can be sustained with budget balance and limited liability. Finally, Proposition 4 shows that efficiency is sustainable with budget balance and limited liability even if player 1 can see the actions taken by all of the other players (I_2 empty).

As in Legros and Matthews [5], let $g(y) = 1/N[y - \sum_{i=1}^{N} v_i(a_i(y)) - W(a^*)]$, where $W(a^*) = y^* - \sum_{i=1}^{N} v_i(a_i^*)$ is the welfare associated with the efficient action vector a^*.

PROPOSITION 1 (Legros and Matthews [5]). *If $I_1 = \emptyset$, efficiency is sustainable with budget balance and limited liability if and only if $g(y) \leqslant 0$ for all y.*

Proof of Proposition 1. See Legros and Matthews [5], Theorem 1. The sufficiency proof for the case where $N = 2$ is reproduced in Lemma 1 below. The necessity proof, slightly adapted, is used in the proof of Lemma 2. ▮

PROPOSITION 2. *If $N = 2$ and $I_2 = \emptyset$, then efficiency is sustainable with budget balance and limited liability if and only if $g(y) \leqslant 0$ for all y.*

Proposition 2 is proved using two lemmas.

LEMMA 1 (Legros and Matthews [5]). *Suppose $N = 2$. If $g(y) \leqslant 0$, then efficiency is sustainable with budget balance and limited liability.*

Proof of Lemma 1. Define the sharing rule

$$z_i(y, R) = \frac{y^*}{2} - v_i(a_i^*) + v_i(a_i(y)) + g(y).$$

If player i chooses action a_i^*, his net utility is

$$\frac{y^*}{2} - v_i(a_i^*). \tag{1}$$

If player i chooses action $a_i(y) \neq a_i^*$, his net utility is

$$\frac{y^*}{2} - v_i(a_i^*) + g(y). \tag{2}$$

Noting that (1) is larger than (2) whenever $g(y) \leqslant 0$ completes the proof. ▮

LEMMA 2. *Suppose $N = 2$. If efficiency is sustainable with budget balance and limited liability then $g(y) \leqslant 0$ for all y.*

Proof of Lemma 2. Suppose that a_1^*, P^*, a_2^* is an equilibrium. This implies that for player 2:

$$s_2(y(a_1^*, a_2(y)), P^*(a_1^*, a_2(y))) - v_2(a_2(y)) \leqslant s_2(y^*, P^*(a^*)) - v_2(a_2^*). \tag{3}$$

The best response condition for player 1 implies:

$$s_1(y(a_1(y), a_2^*), P^*(a_1^*, a_2(y))) - v_1(a_1(y)) \leqslant s_1(y^*, P^*(a^*)) - v_1(a_1^*). \tag{4}$$

Summing (3) and (4) yields

$$y - \sum_{i=1}^{2} v_i(a_i(y)) \leqslant y^* - \sum_{i=1}^{2} v_i(a_i^*).$$

This proves the lemma. ▮

Proof of Proposition 2. Follows directly from Lemmas 1 and 2. ▮

COROLLARY 1 (Legros and Matthews [5], Corollary 2). *If $I_1 = \varnothing$ or $N = 2$ and each $A_i \subset \mathfrak{R}$, $a_i^* \in int(A_i)$, $v_i()$ and $y()$ are C^1, and for each i the partial derivative $\partial y(a^*)/\partial a_i > 0$ there exists a $y < y^*$ such that $g(y) > 0$. Therefore efficiency is not sustainable.*

Propositions 1 and 2 are essentially impossibility results. They show that if $I_1 = \varnothing$ or $N = 2$, efficiency is sustainable if and only if $g(y) \leqslant 0$ for all y. The intuition behind this condition is made apparent in the proof of Lemma 2. For each player, the best response inequality implies that the utility given to that player if he plays his efficient action must be larger than his utility if he plays some other action. Suppose, for example, that player 2 defects to an action that produces output $y < y^*$. In order for this to yield lower utility, the sharing rule must assign a relatively small payment to player 2 when the output is y. However, if the sharing rule is required to balance, decreasing the payment to player 2 when he defects and the output is y entails increasing the reward to player 1 when he mimics player 2's defection by choosing the action $a_1(y)$ and reporting as if player 2 had defected. To determine when neither of these defections is profitable, sum the best response inequalities over both players. This yields the requirement that the aggregate incentive to defect must be smaller than the surplus when the efficient action is played. Why is this so? If it were not, then in order to make the equilibrium payments large enough to keep any player from defecting, the total payments when the efficient action vector is played would be larger than the total output, violating budget balance.

A similar intuition applies in the case considered in Proposition 1, where all partners observe only their own actions. In this case the sharing rule cannot determine which player defected when the output is not y^*. Hence it must punish all players simultaneously. If the condition $g(y) \leqslant 0$ does not hold, then there is some output where the gains from deviation are so large that it would require payments in excess of the efficient output when players play their efficient actions in order to remove the temptation to deviate.

Also noteworthy is the fact that, as Legros and Matthews show, whenever $g(y) \leqslant 0$ for all y, efficiency is sustainable by a sharing rule that does not depend on the report made by player 1. Finally, Corollary 1 shows that differentiability is sufficient for the existence of a y with $g(y) > 0$, and consequently for efficiency not to be sustainable. Hence while efficiency is sustainable when $g(y) \leqslant 0$ for all y, there are a wide variety of environments where this condition will not hold, including most of the well-behaved environments considered in economic models.

Propositions 3 and 4 address the case where $g(y) \leqslant 0$ for all y does not hold. Proposition 3 states that if both I_1 and I_2 are non-empty (implying $N \geqslant 3$), then efficiency is sustainable with budget balance and limited liability. Proposition 4 states that if $N \geqslant 3$ and I_2 is empty, efficiency is sustainable with budget balance and limited liability. Together, Propositions 3 and 4 show that whenever $N \geqslant 3$ and player 1 can observe at least one additional player, efficiency is sustainable with budget balance and limited liability.

The following lemma will be used in the proof of Proposition 3.

LEMMA 3. *For every player i, there is no action a_i such that $y(a_i, a^*_{-i}) > y(a^*)$, and for $s \in (0, 1)$, $sy(a_i, a^*_{-i}) - v_i(a_i) \geqslant sy(a^*) - v_i(a^*_i)$.*

Proof of Lemma 3. $y(a_i, a^*_{-i}) > y(a^*)$ implies that $(1-s)\, y(a_i, a^*_{-i}) > (1-s)\, y(a^*)$. Adding this to $sy(a_i, a^*_{-i}) - v_i(a_i) \geqslant sy(a^*) - v_i(a^*_i)$ yields that $y(a_i, a^*_{-i}) - v_i(a_i) > y(a^*) - v_i(a^*_i)$. Subtracting $\sum_{j \neq i} v_j(a^*_j)$ from both sides yields that $y(a_i, a^*_{-i}) - \sum_{j \neq i} v_j(a_j) - v_i(a_i) > y(a^*) - \sum_j v_j(a^*_j)$, which contradicts that a^* is the efficient action vector. ∎

Intuitively, this lemma states that the efficient output is on the "diminishing returns" part of the players' utility functions. This implies that players will not want to unilaterally deviate to an action that results in an output that is larger than the efficient output.

PROPOSITION 3. *If $N \geqslant 3$ and I_1 and I_2 are both nonempty, efficiency is sustainable with budget balance and limited liability. That is, there exists a sharing rule $z(y, R)$ that is balanced and imposes limited liability and a* *reporting rule* P^* *such that the strategies* $a_i = a_i^*$ *for players other than* 1 *and* (a_1^*, P^*) *for player* 1 *form a perfect Bayesian equilibrium*[1] *of this game.*

Proof of Proposition 3. Since (A0) holds, choose $\{s_i\}_{i=1}^{N}$ and $c \in (0, 1)$ such that $s_i > 0$, $\sum_{i=1}^{N} s_i = 1$, and the following condition holds:

$$(\text{C1}) \quad \begin{cases} \text{(i)} & s_1 + c < 1 & \\ \text{(ii)} & s_1 y^* - v_1(a_1^*) > 0 & \\ \text{(iii)} & \left(s_i - \dfrac{c}{|I_1|}\right) y(a^*) - v_i(a_i^*) > 0 & i \in I_1 \\ \text{(iv)} & s_j y(a^*) - v_j(a_j^*) > 0 & j \in I_2. \end{cases}$$

Consider the following sharing rule, $z(y, R)$, which assigns a share of the output $z_i(y, R)$ to each player based on the realized output y and 1's report $R \in \{G, B\}$.

$$z_1(y, R) = \begin{cases} (s_1 + c)\, y & R = G \quad y \geqslant y^* \\ cy & R = G \quad y < y^* \\ s_1 y + cy^* & R = B \quad y \geqslant y^* \\ cy^* & R = B \quad y < y^* \end{cases}$$

$$z_i(y, R) = \begin{cases} \left(s_i - \dfrac{c}{|I_1|}\right) y & R = G \quad y \geqslant y^* & \\ \dfrac{(1-c)}{|I_1|}\, y & R = G \quad y < y^* & i \in I_1 \\ 0 & R = B \quad y \geqslant y^* & \\ 0 & R = B \quad y < y^* & \end{cases}$$

$$z_j(y, R) = \begin{cases} s_j\, y & R = G \quad y \geqslant y^* & \\ 0 & R = G \quad y < y^* & \\ s_j\, y + \dfrac{(\sum_{i \in I_1} s_i)\, y - cy^*}{N - |I_1| - 1} & R = B \quad y \geqslant y^* & j \in I_2 \\ s_j\, y + \dfrac{(\sum_{i \in_1} s_i) + s_1)\, y - cy^*}{N - |I_1| - 1} & R = B \quad y < y^*. & \end{cases}$$

[1] Perfect Bayesian equilibrium is considered instead of sequential equilibrium because sequential equilibrium is defined for finite games only, and the possibility of uncountable action sets is allowed here. In particular, the notion of consistent beliefs employed in sequential equilibrium fails to apply in games with general action sets. However, if the action sets used here were finite, then the equilibrium discussed would be sequential. Furthermore, the beliefs used here would likely satisfy any extension of the definition of consistent beliefs to general action sets.

This is a balanced sharing rule with limited liability.

In sharing rule z, note that y is the realized output of the production function, while $y^* \equiv y(a^*)$ is the efficient output.

Define 1's reporting rule P^* as

$$P^*(a_1, a_{I_1}) = \begin{cases} B & \text{if} \quad y(a_1, a_{I_1}, a^*_{I_2}) < y^* \\ G & \text{if} \quad y(a_1, a_{I_1}, a^*_{I_2}) \geqslant y^*. \end{cases}$$

The rule P^* is designed to be a best response along the path where the players in I_2 play their efficient actions. Conditional on an action vector $(a_1, a_{I_1}, a^*_{I_2})$, player 1's preferences only depend on how $cy(a_1, a_{I_1}, a^*_{I_2})$ compares to cy^*, and the former at least as large as the latter whenever $y(a_1, a_{I_1}, a^*_{I_2}) \geqslant y^*$. Hence P^* specifies a best report.

To prove that the above strategy profile is a perfect Bayesian equilibrium of this game, it suffices to specify consistent beliefs and show that the strategy is sequentially rational given these beliefs. Since players other than player 1 move only once and with no information, we need only specify beliefs at each of player 1's information sets. Let $\mu_{(a_1, a_{I_1})}(a)$ be the belief probability assigned to the node associated with the action vector a conditional on reaching information set (a_1, a_{I_1}). We specify beliefs as

$$\mu_{(a_1, a_{I_1})}(a_1, a_{I_1}, a^*_{I_2}) = 1.$$

That is, player 1 believes with probability 1 that if information set (a_1, a_{I_1}) is reached, player 1 and all players in I_1 acted as observed, and the players in I_2 played their efficient actions. All other nodes are assigned 0 probability of being played. These beliefs are consistent in the sense of perfect Bayesian equilibrium, since they agree with Bayes' rule along the equilibrium path.

Next, sequential rationality is established in three steps.

Step 1. Consider player 1. Recall that P^* was designed to be an optimal reporting rule at each of 1's information sets, given that all players in I_2 play their efficient actions. Hence P^* is sequentially rational given the beliefs μ as specified above. It remains to show that player 1's choice of action is also sequentially rational given that he reports according to P^*. Player 1's choice of a_1 falls into three cases.

Case 1a. $a_1 = a_1^*$. In this case, $y(a_1, a^*_{-1}) = y^*$ and 1 reports G, getting payoff $(s_1 + c)\, y^* - v_1(a_1^*)$.

Case 1b. $a_1 \neq a_1^*$ and $y(a_1, a^*_{-1}) < y^*$. In this case 1 reports B, getting payoff $cy^* - v_1(a_1)$.

Case 1c. $a_1 \neq a_1^*$ and $y(a_1, a^*_{-1}) > y^*$. In this case 1 reports G, getting payoff $(s_1 + c)\, y(a_1, a^*_{-1}) - v_1(a_1)$.

By (C1), $s_1 y^* - v_1(a_1^*) > 0 \geqslant -v_1(a_1)$. Hence $a_1 = a_1^*$ is preferred to any action in case (1b). Lemma 3 proves that no action in case (1c) can have a higher payoff than $a_1 = a_1^*$. Thus $a_1 = a_1^*$ is optimal for 1 given that he plays P^* (which is sequentially rational independent of what action 1 plays) and the other players play their efficient actions. Hence (a_1^*, P^*) is a sequentially rational best response for player 1.

Step 2. Consider player $i \in I_1$. Player i's strategy choice falls into three cases:

Case 2a. $a_i = a_i^*$. In this case i's payoff is $(s_i - (c/|I_1|))\, y^* - v_i(a_i^*)$, since 1 reports G and $y = y^*$.

Case 2b. $a_i \neq a_i^*$ and $y(a_i, a_{-i}^*) < y^*$. In this case i's payoff is $-v_i(a_i)$, since 1 reports B.

Case 2c. $a_i \neq a_i^*$ and $y(a_i, a_{-i}^*) > y^*$. In this case i's payoff is $(s_i - (c/|I_1|))\, y(a_i, a_{-i}^*) - v_i(a_i)$ since 1 reports G.

Since $(s_i - (c/|I_1|))\, y^* - v_i(a_i^*) > 0 \geqslant -v_i(a_i)$ by (C1), choosing $a_i = a_i^*$ is preferred to any action in case (2b). Lemma 3 shows that no action in case (2c) can have a payoff that is higher than that of $a_i = a_i^*$. Hence $a_i = a_i^*$ is i's unique best response.

Step 3. Finally, consider player $j \in I_2$. Player j's actions can be divided into the same three cases.

Case 3a. $a_j = a_j^*$. In this case, $y(a_j, a_{-j}^*) = y^*$ and 1 reports G, yielding payoff $s_j y^* - v_j(a_j^*)$.

Case 3b. $a_j \neq a_j^*$ and $y(a_j, a_{-j}^*) < y^*$. In this case 1 reports G, yielding payoff $-v_j(a_j)$.

Case 3c. $a_j \neq a_j^*$ and $y(a_j, a_{-j}^*) > j^*$. In this case 1 reports G, yielding payoff $s_j y(a_j, a_{-j}^*) - v_j(a_j)$.

The same arguments show that $a_j = a_j^*$ is a best response for player j.
Hence the strategy profile $a_i = a_i^*; i \neq 1, (a_1^*, P^*)$ is a perfect Bayesian equilibrium of this game. ∎

COROLLARY 2. *If $N \geqslant 3$ and $|I_1| = 1$ (meaning player 1 can only see the action of one other player), efficiency is sustainable with budget balance and limited liability.*

The intuition behind Proposition 3 is relatively simple. The quantities cy and cy^* can be thought of as payments made by the players in I_1 to player 1. The former is smaller than the latter whenever the realized output is less than the efficient output. Hence whenever a player in I_1 plays an action that is too small, player 1 reports B. When a player in I_1 plays an action that is larger than the efficient action, player 1 reports G. Since Lemma 3 ensures that no player in I_1 will want to choose an action yielding output larger than y^*, any such player's choice reduces to choosing the efficient action, yielding a report of G and a payoff of $(s_i - (c/|I_1|))\, y^* - v_i(a_i^*)$, or choosing an action yielding an output smaller than y^*, leading to a report of B and a payoff of $-v_i(a_i)$. Since the former is positive, player i chooses a_i^*. In this way the reporting rule P^* prevents the players in I_1 from cheating. Once we know that no player in I_1 will cheat, we can impose group penalties on the remaining players whenever the output is too small and player 1's report is G. In this case, I_1 absorbs the output taken from the other players, maintaining budget balance. In short, it is player 1's threat to report B if a player in I_1 cheats that causes them to choose their efficient actions, and the ability of the players in I_1 to absorb the group penalty imposed on I_2 when the output is too small and the report is G that allows efficiency to be sustained and budget balance maintained. Furthermore, this is all done while maintaining limited liability.

Corollary 2 states that if $N \geqslant 3$ and player 1 can only observe his own action and the action taken by a single other player, efficiency is sustainable. This illustrates how little additional information is needed to sustain efficiency. All that is really needed is that there be some player who is observed by player 1 and some player who is not. The unobserved player serves as a sink to absorb the output when the observed player must be punished. This yields further intuition for why efficiency cannot in general be sustained when $N = 2$. Since there is no player to absorb penalties imposed on player 2 except player 1, budget balance implies that in punishing player 2, player 1 is simultaneously rewarding himself, making it impossible to overcome player 1's moral hazard problem.

Proposition 4 shows that efficiency is sustainable when player 1 can observe the actions taken by all of the other players (I_2 empty).

PROPOSITION 4. *Suppose $N \geqslant 3$ and $I_2 = \varnothing$. Efficiency is sustainable with budget balance and limited liability. That is, there exists a sharing rule $x(y, R)$ that is balanced and imposes limited liability and a reporting rule P^* such that the strategies $a_i = a_i^*$ for players other than 1 and (a_1^*, P^*) for player 1 form a subgame perfect equilibrium[2] of this game.*

Proof of Proposition 4. Since ($A0$) holds, choose $\{s_i\}_{i=1}^N$ and $c \in (0, 1)$ such that $s_i > 0$, $\sum_{i=1}^N s_i = 1$, and the following condition holds.

[2] Subgame perfect equilibrium is used here since all of the nodes at which player 1 must report are singletons.

$$\text{(C2)} \quad \begin{cases} \text{(i)} \quad s_1 + c < 1 \\ \text{(ii)} \quad s_1\, y(a^*) - v_1(a_1^*) > 0 \\ \text{(iii)} \quad \left(s_i - \dfrac{c}{N-1}\right) y(a^*) - v_i(a_i^*) > 0 \qquad i \neq 1 \end{cases}$$

Consider the following sharing rule, $x(y, R)$, in which the range of player 1's reports is the set of all subsets of I_1. Hence 1's reporting rule is a map from action vectors a to $B \subset I_1$, and $x_i(y, R)$ is the share of the output assigned to player i.

$$x_1(y, R) = \begin{cases} (s_1 + c)\, y & y \geqslant y^* \quad R = \varnothing \\ cy & y < y^* \quad R = \varnothing \\ s_1\, y + cy^* & y \geqslant y^* \quad R \neq \varnothing \\ cy^* & y < y^* \quad R \neq \varnothing \end{cases}$$

$$x_i(y, R) = \begin{cases} \left(s_i - \dfrac{c}{N-1}\right) y & y \geqslant y^* \quad R = \varnothing \\ \left(s_i + \dfrac{s_1 - c}{N-1}\right) y & y < y^* \quad R = \varnothing \\ s_i\, y - \left(\dfrac{c}{N-1}\right) y^* & y \geqslant y^* \quad i \in B \\ 0 & y < y^* \quad i \in B \\ s_i\, y - \left(\dfrac{c}{N-1}\right) y^* & y \geqslant y^* \quad i \notin B \\ s_i\, y + \dfrac{(s_1 + \sum_{j \in B} s_j)\, y - cy^*}{N - |B| - 1} & y < y^* \quad i \notin B \end{cases} \qquad i \neq 1.$$

This is a balanced sharing rule with limited liability.

Specify the following reporting rule for player 1.

$$P^*(a) = \begin{cases} \varnothing & y(a) \geqslant y^* \\ B = \{i \mid i \in B \text{ iff } a_i \neq a_i^*\} & y(a) < y^*. \end{cases}$$

Intuitively, whenever the realized output is too small, player 1 announces the set of players B who did not play their efficient actions. The proof proceeds in two steps and is similar to the proof of Proposition 3.

Step 1. Consider player 1. Since 1 reports $P^*(a) = \varnothing$ whenever $y(a) \geqslant y^*$ and $P^*(a) = B \neq \varnothing$ whenever $y(a) < y^*$, $P^*(a)$ is an optimal reporting rule at each information set a.

If 1 chooses action a_1^*, he gets utility $(s_1+c)\, y^* - v_1(a_1^*)$. If he chooses a_1 such that $y(a_1, a_{-1}^*) < y^*$, he reports $P^*(a_1, a_{-1}^*) = B \neq \emptyset$ and gets payoff $cy^* - v(a_1)$. By (C2), the former is always larger than the latter. If he chooses a_1 such that $y(a_1, a_{-1}^*) > y^*$, he gets $(s_1+c)\, y(a_1, a_{-1}^*) - v_1(a_1)$. Lemma 3 ensures that no such action can yield higher utility than $a_1 = a_1^*$. Hence $a_1 = a_1^*$, $P^*(a)$ is a best response to a_{-1}^*.

Step 2. Consider player $i \neq 1$. If i plays action a_i, he gets utility $(s_i - (c/N-1))\, y^* - v_i(a_i^*)$. If i plays action a_i such that $y(a_i, a_{-i}^*) < y^*$, he gets utility $-v_i(a_i)$, since 1 reports $i \in B$. Since $(s_i - (c/N-1))y^* - v_i(a_i^*)$ is assumed positive in (C2), a_i^* is preferred to any action such that $y(a_i, a_{-i}^*) < y^*$. If i plays action a_i such that $y(a_i, a_{-i}^*) > y^*$, he gets payoff $(s_i - (c/N-1))\, y(a_i, a_{-i}^*) - v_i(a_i^*)$. Lemma 3 ensures that no such action can yield higher utility than $a_i = a_i^*$. This completes the proof. ∎

The intuition for how this mechanism works is the same as in the mechanism used to prove Proposition 3. Indeed, the mechanism used in Proposition 4, slightly adapted, could have been used to prove Proposition 3. However, as compared to the mechanism used in Proposition 3, this mechanism involves reports with a more complex range. In the Proposition 4 mechanism, player 1 is indifferent between reporting any non-empty subset B of I_1 whenever $y < y^*$. This is because, conditional on his action, the reward player 1 gets for making report R depends only on the size of the output (which 1 knows), and whether his report is the empty set or some non-empty subset of I_1. Hence, conditional on $a_1 = a_1^*$, the truthful reporting rule "$i \in B$ if and only if $a_i \neq a_i^*$" is one optimal reporting rule for player 1. There are also many others. However, when 1 plays this reporting rule, it reduces the decisions of the remaining players to choosing between playing a_i^* and getting a positive utility and playing some other $a_i \neq a_i^*$ and getting a negative utility, and this is sufficient to sustain efficiency.

Before concluding, several general comments about the mechanisms employed here are in order. First, the mechanisms employed in Propositions 3 and 4 satisfy a slightly stronger version of limited liability than the one used in the original definition. Earlier, limited liability was said to hold whenever there exists a k such that for all i, y, and R, $z_i(y, R) \geqslant k$. That is, we required all payments under the sharing rule to be larger than k. The sharing rules used in Propositions 3 and 4 are such that for any $k < 0$, $z_i(y, R) \geqslant k$ for c chosen sufficiently small. In other words, by choosing c sufficiently small, any negative payments the sharing rules impose can be made arbitrarily small. Furthermore, none of the results in this paper depend on c being large enough. Thus, in effect, the mechanisms do not depend on any assumptions about the initial wealth of the partners.

Finally, consider the vulnerability to coalition defections of the mechanisms used here. Indeed, both mechanisms are vulnerable to coalition defections if side payments are permitted. However, if side payments are not permitted, then there is no coalition deviation to a different action vector that is weakly preferred by all members of the coalition.[3] This arises from the fact that in forcing the mechanism to be balanced, we necessarily offset a reward to one agent with punishments to other agents. The fact that agents' action choices are opposed to each other in this manner accounts for the lack of coalition-preferred deviations.

4. CONCLUSION

The results in this paper characterize when efficiency is sustainable with budget balance and limited liability in partnerships with joint monitoring. The main positive result is that efficiency is sustainable with budget balance whenever there are at least three players and player 1 can see the action taken by at least one other player. The proofs of Propositions 3 and 4 present simple balanced mechanisms that sustain efficiency, although there almost certainly are others.

The mechanisms employed in sustaining efficiency, in conjunction with the Legros and Matthews [5] result, point toward a general method that may be useful in solving a wide variety of mechanism design problems where budget balance is a concern. First, a mechanism is designed that aligns the incentives of some of the players on the set of outcomes that can be reached through unilateral deviation by one of these players. Once it is known that these players will behave as desired, they can be used to absorb group penalties imposed on the remaining players whenever the outcome could not have arisen through unilateral deviation by the controlled group. This implements the desired action profile. In this paper, player 1's reporting rule aligns the incentives of the players in I_1 when $y \geqslant y^*$ and $R = G$ or $y < y^*$ and $R = B$, and group penalties imposed when $y < y^*$ and $R = G$ discipline the remaining players. In Legros and Matthews [5], player 1 is exactly compensated for his cost of effort whenever the output is such that he could have caused it by unilateral deviation. His moral hazard problem eliminated, player 1 is then used to absorb group penalties imposed on the other players whenever the realized output could not have resulted from unilateral deviation by player 1. This implements the (approximately) efficient outcome. The success of these two approaches suggests that the general technique of controlling the incentives of one player and then using that player to absorb group penalties may be a useful tool in designing balanced mechanisms to solve a wide variety of problems.

[3] I thank the referee for this observation.

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