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The equivalence of price and quantity competition with delegation

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In a two-stage differentiated-products oligopoly model, profit-maximizing owners first choose incentive schemes in order to influence their managers' behavior. In the second stage, the managers compete either both in prices, both in quantities, or one in price and the other in quantity. If the owners have sufficient power to manipulate their managers' incentives, the equilibrium outcome is the same regardless of how the firms compete in the second stage. If demand is linear and marginal cost is constant, basing the manager's objective function on a linear combination of the firm's profit and its rival's profit is sufficient for the equivalence result.

1. Introduction

■ Economists have long recognized that the outcome of oligopolistic competition when firms set prices differs from the outcome when firms set quantities. Cournot (1838) and Bertrand (1883) demonstrate that when firms produce identical products, quantity competition results in positive price-cost margins, while price competition results in price being driven to marginal cost. More recently, Singh and Vives (1984) show that when firms produce differentiated products with linear demand and constant marginal cost, price competition leads to higher quantities and lower prices than quantity competition.[1]

The difference between price and quantity competition arises from the fact that price-setting behavior by one's opponent has different strategic implications for a firm's profit-maximization problem than quantity-setting behavior. In this article we argue that in two-stage delegation games where the owner of each firm exercises sufficient control over the behavior of its manager, the equilibrium outcomes under price and quantity competition involve the same final prices and quantities. In particular, when demand is linear and output is produced at constant marginal cost, a firm's ability to compensate its manager based on a linear combination of its own profit and

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[1] With more general demand, Singh and Vives (1984) show that when goods are substitutes, quantity competition leads to larger equilibrium prices than does price competition, and when goods are complements, quantity competition leads to smaller equilibrium quantities than does price competition. Thus the ordering results apply to whichever variable exhibits strategic complementarity.

its rival's profit is sufficient for the equilibria under price and quantity competition to coincide. Furthermore, we characterize sufficient conditions for the equivalence result to hold in more general environments and show that the equivalence result holds even when the delegation game has multiple equilibria.

Singh and Vives (1984) and Cheng (1985) study the equilibrium outcome disparity between price and quantity competition in differentiated-products environments without delegation.[2] Regardless of whether a firm competes by setting price or quantity, its profit-maximizing behavior reduces to choosing a price-quantity pair off the residual demand curve defined by its opponent's strategy choice. The residual demand curve that a firm faces is more elastic when the firm believes its opponent holds its price fixed than when it believes its opponent holds its quantity fixed. Hence it has stronger incentives to move toward lower prices and higher quantities under price competition than under quantity competition.[3]

When the differentiated-products competition takes place as part of a delegation game, the elasticity of residual demand that a firm's manager perceives can be manipulated by the firm's owner through the incentive scheme she sets for the manager.[4] Various authors have considered the impact of manipulating the behavior of agents in duopoly games to gain a strategic advantage. For example, Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) each consider two-stage delegation games where in the first stage a profit-maximizing owner chooses an incentive scheme for her manager based on a weighted difference of revenue and cost, and in the second stage the manager competes in an oligopoly game. Fumas (1992) and Miller and Pazgal (1997) consider two-stage games in which the manager's incentive scheme is based on a weighted sum of the firm's own profit and its rival's profit. In each case, by manipulating the incentive parameters it chooses for its manager, the firm is able to affect the manager's behavior and commit to strategies in the product market that are not profit maximizing, thereby gaining a strategic advantage in the two-stage game. The main result of this article is that if, in a delegation game, the set of incentive parameters available to the owner is rich enough (i.e., any behavior by the manager that can be induced in the price-setting game can also be induced in the quantity-setting game), then the equilibrium prices and quantities will be the same regardless of whether the firms compete by both setting prices, by both setting quantities, or by one setting price and the other setting quantity. Section 2 illustrates the intuition of the result in a simple example. Section 3 presents the general linear model, states the result (with derivation contained in the Appendix), and discusses its geometry. Section 4 proves the equivalence result in a more general setting. Section 5 concludes.

2. An illustration

■ We begin with a simple example that describes the two-stage delegation game and illustrates the method of deriving its subgame-perfect Nash equilibrium. The derivation in the general linear case follows the same path. Each firm has an owner and a manager. The manager makes the actual choice of the firm's strategic variable, but the owner is the residual claimant on the firm's profits. The owner compensates the manager based on a linear combination of the firm's own profit and its rival's profit. For simplicity, we assume the marginal cost of production for either firm is constant and equal to zero, and that each firm has no fixed costs.

The products offered by the firms are differentiated, and, provided all prices and quantities are nonnegative, demand for each is given by

$$q_i = 1 - p_i + zp_j,$$

[2] For a concise survey of results in this area, see Vives (1999).

[3] See Singh and Vives (1984) and Cheng (1985) for a more complete discussion of this point.

[4] For expositional ease, throughout the article we treat owners as female, managers as male, and firms as neuter.

where q_i and p_i refer to quantities and prices chosen by firm $i \in \{1, 2\}$, and $0 < |z| < 1$ for stability of the demand system.[5] If $z > 0$, the goods are gross substitutes. If $z < 0$, they are gross complements.

Consider the delegation game where in the second stage firms compete by setting prices. Let θ_i^p be the weight owner i places on the profit of the other firm in her manager's compensation scheme. In this environment, the managers' objective functions are given by

$$m_i^p = \textit{Own profit} + \theta_i^p(\textit{Firm } j\textit{'s profit}) \tag{1}$$
$$= \left(1 - p_i + zp_j\right) p_i + \theta_i^p \left(1 - p_j + zp_i\right) p_j. \tag{2}$$

Since such schemes involve comparison of the firm's own profit with its rival's, we will refer to them as relative performance schemes. Differentiating (2) with respect to p_i, setting the result equal to zero, and solving for p_i yields the manager's reaction function:

$$p_i\left(p_j\right) = \frac{1}{2}\left(1 + z\left(1 + \theta_i^p\right) p_j\right). \tag{3}$$

Solving system of equations (3) implies second-stage equilibrium prices:

$$p_i = \frac{2 + z\left(1 + \theta_i^p\right)}{4 - z^2\left(1 + \theta_i^p + \theta_j^p + \theta_j^p\theta_i^p\right)}. \tag{4}$$

Let $x_i = (1 - p_i + zp_j)p_i$ be the profit earned by firm i. Evaluating at the equilibrium prices from (4) gives the following expression for profit as a function of the incentive parameters chosen by the two firms:

$$x_i = \left(2 + z\left(1 + \theta_i^p\right)\right) \frac{2 + z - z\theta_i^p\left(z + z\theta_j^p + 1\right)}{\left(4 - z^2\left(1 + \theta_i^p + \theta_j^p + \theta_j^p\theta_i^p\right)\right)^2}.$$

Differentiating with respect to θ_i^p, setting the result equal to zero, and solving for θ_i^p yields the owners' reaction functions:

$$\theta_i^p = \left(1 + \theta_j^p\right)(z + 2)\frac{z}{z\left(z + z\theta_j^p - 2\theta_j^p\right) - 2z - 4}. \tag{5}$$

The solution to system of equations (5) that also fulfills the firms' second-order conditions implies equilibrium incentive parameter values $\theta_i^p = z/(2-z)$. The resulting equilibrium prices, quantities, and profits are

$$p_i^p = \frac{1}{4}\left(\frac{z - 2}{z - 1}\right), \tag{6}$$

$$q_i^p = \frac{1}{4}z + \frac{1}{2}, \tag{7}$$

and

$$x_i^p = \frac{1}{16}\left(\frac{z^2 - 4}{z - 1}\right). \tag{8}$$

[5] Here and throughout the article we will use i to denote a generic firm, $i \in \{1, 2\}$, and $j = 3 - i$ to denote the other firm.

Next, we repeat the process for the case where firms compete by setting quantities. Letting θ_i^q be the weight the owner of firm i puts on the profit of firm $j \neq i$ in its manager's compensation scheme, the manager's objective function is given by

$$m_i^q = \left(\frac{1+z-q_i-zq_j}{1-z^2}\right) q_i + \theta_i^q \left(\frac{1+z-q_j-zq_i}{1-z^2}\right) q_j$$

which implies reaction function

$$q_i\left(q_j\right) = \frac{1+z-z\left(1+\theta_i^q\right) q_j}{2}. \tag{9}$$

Completing the computation of the equilibrium of the delegation game when the managers compete in quantities shows that in equilibrium $\theta_i^q = -z/(z+2)$, and that the equilibrium prices and quantities are the same as in (6)–(8), where the firms compete by setting prices.

In this example, the equilibrium prices and quantities do not depend on whether the firms compete by setting prices or quantities. This provides the first instance of our equivalence result. To gain an initial intuition, let the superscript pnd denote outcome variables in nondelegation price competition, and qnd denote outcome variables in nondelegation quantity competition. Without delegation price competition results in equilibrium prices $p_i^{pnd} = 1/(2-z)$ and quantities $q_i^{pnd} = 1/(2-z)$, while quantity competition results in equilibrium prices $p_i^{qnd} = 1/(1-z)(2+z)$ and quantities $q_i^{qnd} = (1+z)/(2+z)$ for each firm. Thus, in this example, equilibrium prices and quantities of the delegation game fall in between those of nondelegation price and quantity competition in this example. In the delegation game when the goods are substitutes ($0 < z < 1$) and firms compete by setting prices, each manager's equilibrium incentive scheme puts positive weight on the profit of the other firm. Since under this incentive scheme any gain in profit due to lowering his own price is partially offset by the accompanying decrease in the other firm's profit, this makes the manager less aggressive: his optimal reaction to any price charged by the other firm is now higher than it was in the nondelegation game (see (3)). Conversely, when the firms compete in quantities, the equilibrium incentive scheme puts negative weight on the other firm's profit. This tends to make the managers more aggressive, since the gain due to increasing quantity is augmented by the corresponding decrease in the other firm's profit.[6] As can be seen from (9), the effect is that the manager responds to any quantity choice by his rival with a larger quantity than he would in the nondelegation game.

Without delegation, price competition is more aggressive than quantity competition. The effect of delegation is to make price competition less aggressive and quantity competition more aggressive until the manager's behavior is, in fact, the same in the two cases. And, identical behavior leads to identical outcomes. This is the essence of the equivalence result, which is developed more generally in the next two sections.

3. The linear model

■ In this section we develop the equivalence result for delegation games with linear demand and constant marginal cost in which each manager is compensated based on a linear combination of his own firm's profit and his rival's. We present the model, derive the result by direct computation, and provide a geometric analysis. In Section 4, a general proof is given that does not depend on computation of the equilibria.

[6] A similar argument can be made in the case of complementary goods. In delegated price competition owners put negative weight on the other firm's profit. However, since the products are complements this has the effect of making manager less aggressive since the outward shift in the other firm's demand accompanying an increase in price works against the manager in this case. The opposite is true in the case of quantity competition.

Consider the general linear inverse-demand system used by Dixit (1979) and Singh and Vives (1984) given by

$$p_i = \alpha_i - \beta_i q_i - \gamma_i q_j,$$

provided that all prices and quantities are nonnegative. Again, provided that prices and quantities are nonnegative, the corresponding direct demand system is given by

$$q_i = a_i - b_i p_i + z_i p_j, \tag{10}$$

where $a_i = (\alpha_i \beta_j - \alpha_j \gamma_i)/(\beta_i \beta_j - \gamma_i \gamma_j)$, $b_i = \beta_j/(\beta_i \beta_j - \gamma_i \gamma_j)$, and $z_i = \gamma_i/(\beta_i \beta_j - \gamma_i \gamma_j)$. Each firm produces output at constant marginal cost $c_i \geq 0$. We impose the following restrictions on the parameters:

$$\alpha_i > c_i, \quad \beta_i > 0, \quad \gamma_i \neq 0, \quad \text{sign}(\gamma_i) = \text{sign}(\gamma_j), \quad \text{and} \quad \beta_i \beta_j - \gamma_i \gamma_j > 0. \tag{11}$$

If the sign of γ_i (and z_i) is positive then the products produced by the firms are gross substitutes. If the sign is negative then the products are gross complements. Note that $\beta_i > 0$ and $\beta_i \beta_j - \gamma_i \gamma_j > 0$ imply that $b_i > 0$. In addition, we restrict the parameters of the demand system to be such that if firms 1 and 2 charge prices $p_1 = c_1$ and $p_2 = c_2$, consumers demand positive quantities of each product:

$$(\alpha_i - c_i)\beta_j - \left(\alpha_j - c_j\right)\gamma_i > 0. \tag{12}$$

Condition (12) implies that $\alpha_i \beta_j - \alpha_j \gamma_i > 0$, which in turn implies that $a_i > 0$.[7]

The two-stage delegation game proceeds as in the previous section. In the first stage, each owner chooses the weight that it will put on the other firm's profit in her manager's incentive scheme. Thus the owner's objective function remains as in (1), only with (10) replacing the example's demand system. In the second stage the managers compete in the product market. We consider three types of product market competition: both firms set price, both firms set quantity, or one firm sets price and the other firm sets quantity. Propositions 1 and 2 characterize the equilibria of the three games.

Proposition 1. Let $\bar{\theta}_i^{st}$ be the subgame-perfect Nash equilibrium values of the incentive parameters in the delegation game where firm i's second-stage strategic variable is $s \in \{p, q\}$ and firm j's second-stage strategic variable is $t \in \{p, q\}$. These values are given by

$$\bar{\theta}_i^{sp} = \frac{\gamma_i(\alpha_i - c_i)}{2\left(\alpha_j - c_j\right)\beta_i - (\alpha_i - c_i)\gamma_j}$$

$$\bar{\theta}_i^{sq} = \frac{-\gamma_i\left((\alpha_i - c_i)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right)}{\beta_j\left(\left(\alpha_j - c_j\right)\beta_i - (\alpha_i - c_i)\gamma_j\right) + \left(\alpha_j - c_j\right)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}.$$

Proof. See the Appendix.

Proposition 1 states that firm i's equilibrium incentive parameter depends only on whether firm j chooses price or quantity and does not depend on whether its own strategic variable is price or quantity. This stands to reason, since, as Klemperer and Meyer (1986) note for the case of nondelegation games, holding fixed the strategic choice of his opponent, regardless of whether it is a quantity choice or a price choice, a manager's maximization problem amounts to choosing a point off the resulting residual demand curve. Thus it does not matter whether the manager himself chooses price or quantity. On the other hand, as Singh and Vives (1984) note

[7] Although Singh and Vives (1984) consider the case where $\gamma_i = \gamma_j$, assumption (12) is implicit in their model since they embed marginal cost in the alpha terms and assume $\alpha_i \beta_j - \alpha_j \gamma_i > 0$.

in nondelegation games, the residual demand curve that a manager faces differs depending on whether his opponent chooses price or quantity. So an owner will choose incentive parameters to influence her manager's behavior in response to the other firm's strategic variable, which affects behavior, but not in response to her own firm's strategic variable, which does not.

Proposition 1 also implies that if firm i's opponent sets price in the second stage, its own equilibrium incentive parameter has the same sign as γ_i. If firm i's opponent sets quantity in the second stage, its own equilibrium incentive parameter has the opposite sign from γ_i. Thus, as in the example, owners use their ability to manipulate their managers' incentives to make them act less aggressively when the other manager sets price and more aggressively when the other manager sets quantity.

Using the equilibrium values of the incentive parameters to compute the equilibrium prices and quantities in each case yields our main result for the linear model.

Proposition 2. Regardless of whether both firms set prices, both firms set quantities, or one firm sets price and the other sets quantity, the subgame-perfect Nash equilibrium values of the prices and quantities are given by[8]

$$\bar{q}_i = \frac{1}{4}\left(\frac{\beta_i\left((\alpha_i - c_i)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right) + (\alpha_i - c_i)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}{\beta_i\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}\right)$$

and

$$\bar{p}_i = \frac{2(\alpha_i - c_i)\beta_j - \left(\alpha_j - c_j\right)\gamma_i}{4\beta_j} + c_i.$$

Proof. See the Appendix.

However firms compete, the equilibrium prices and quantities in the delegation game are the same, although the incentive parameters, as given in Proposition 1, may differ. We delay discussion of the intuition behind the equivalence result to the next subsection, where we give a geometric argument for why the equilibria under price and quantity competition must coincide.

The two corollaries below compare the equilibrium prices and quantities of the delegation game to nondelegation price (pnd) and quantity (qnd) competition.

Corollary 1. The equilibrium prices are such that

(i) $p_i^{qnd} > \bar{p}_i$.

(ii) $\bar{p}_i > p_i^{pnd}$ if and only if $(\alpha_i - c_i)/(\alpha_j - c_j) > -\gamma_i/2\beta_j$.

Corollary 2. The equilibrium quantities are such that

(i) $q_i^{pnd} > \bar{q}_i$.

(ii) $\bar{q}_i > q_i^{qnd}$ if and only if $(\alpha_i - c_i)/(\alpha_j - c_j) > 3\beta_i\gamma_i/(2\beta_i\beta_j + \gamma_i\gamma_j) = 1/(2\beta_j/3\gamma_i + \gamma_j/3\beta_i)$.

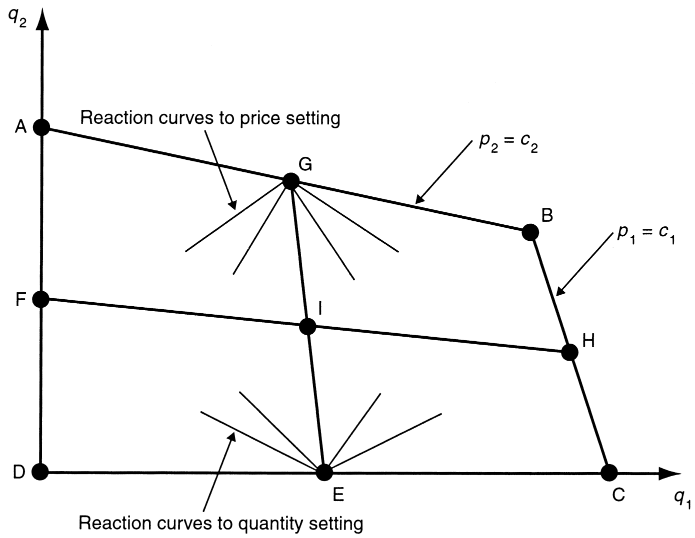
Proof. See the Appendix.

Corollaries 1 and 2 show that it need not be the case that the equilibrium prices and quantities in the delegation game fall between those of the nondelegation price and quantity games. But whenever the goods are substitutes, $p_i^{qnd} > \bar{p}_i > p_i^{pnd}$, and whenever the goods are complements, $q_i^{pnd} > \bar{q}_i > q_i^{qnd}$.[9] Further, (11) and (12) imply that it cannot be that $p_i^{pnd} \geq \bar{p}_i$ or $q_i^{qnd} \geq \bar{q}_i$ simultaneously for both players.

[8] Conditions (11) and (12) are sufficient for the equilibrium quantities and profits to be positive, although these conditions are not necessary. Necessary and sufficient conditions are provided in the Appendix.

[9] The fact that the ordering of the prices and quantities is not always the same makes comparison of profits under the various scenarios quite complicated. The complete analysis is available from the authors upon request.

FIGURE 1



Although our ordering results are incomplete, they correspond to the results in Singh and Vives (1984) in that the side of the market with a necessary ordering is the side that exhibits strategic complementarity, i.e., prices when the goods are substitutes and quantities when the goods are complements. Nevertheless, while the ordering results may fail to hold in some circumstances, the equivalence result holds for any specification of the parameters satisfying the basic requirements in (11) and (12).

□ **The geometry of the linear model.** We now provide a geometric derivation of the equivalence result in the linear case. Figure 1 depicts the equilibrium of the delegation game in quantity space.[10] Line segments AB and BC represent the translation into the quantity space of the lines $p_2 = c_2$ and $p_1 = c_1$, respectively. Consider the quantity-setting delegation game. If manager j chooses $q_j = 0$ in the second stage, it earns zero profit. Hence manager i's best response to $q_j = 0$ is the same regardless of the weight put on the other firm's profit in his objective function. As the weight put on the other firm's profit is varied, manager i's reaction function pivots through these points, which are denoted E for manager 1 and F for manager 2. Similarly, in price competition the manager's best response will be the same whenever the rival manager chooses $p_j = c_j$, regardless of the weight put on the other firm's profit. Denote these best-response points by G for manager 1 and H for manager 2.

Consider the case where manager 2 chooses $q_2 = 0$ in the quantity-setting game. If manager 1 chooses $q_1 = 0$ in response, firm 1 earns zero profit (point D). If he increases his quantity until price equals c_1, firm 1 also earns zero profit (point C). Along segment CD, firm 1's profit follows a quadratic function. Hence its profit is maximized at the midpoint, point E. Similarly, points F, G, and H are the midpoints of segments AD, AB, and BC, respectively. Since the segments connecting the midpoints of opposite sides of a quadrilateral bisect each other, point I lies halfway between points F and H and points E and G.

We now argue that point I must be the equilibrium quantity vector in the delegation game. Suppose that owner 2 chooses reaction function FH. This can be done by setting either price or quantity, and in response to either price setting or quantity setting. Owner 1 makes her choice of incentive parameters in order to choose the point off FH that maximizes her profit. Since firm 1's

[10] A key to the prices and quantities associated with each labelled point in the figure can be found in Table A1 in the Appendix.

profit is a function of q_1, q_1^2, and q_1q_2 and FH is linear in q_2, firm 1's profit follows a parabola when projected along segment FH. The maximum of a parabola is halfway between its roots, and firm 1 earns zero profit at points F (where it sells zero quantity) and H (where it charges $p_1 = c_1$). Hence firm 1's profit must be maximized at point I, and thus reaction curve EG is owner 1's best response to FH. The same argument shows that FH is a best response to EG, and thus that point I must be the equilibrium of the delegation game.

4. Equivalence in general environments

■ We now take a more general approach to the equivalence result. In particular, we drop the assumption that the demand system is linear, although we will maintain the assumption that it is invertible. In addition, we allow owners to choose general incentive parameters instead of restricting them to relative performance incentive schemes. Three types of second-stage competition are considered: both firms compete in prices, both firms compete in quantities, and one firm competes in prices and the other in quantities. Throughout the section we assume that each type of competition has a unique equilibrium. The extension of the result to the case of multiple equilibria is addressed in Remark 2 and illustrated at the end of this section.

In the following derivation, we work with a generalization of the reaction function, which we call the *outcome set.* The outcome set for manager i consists of all price-quantity quadruples that

(i) are consistent with the demand system

(ii) represent a utility-maximizing choice for manager i given the strategic choice of manager j and the incentive parameters chosen by owner i.

The outcome set represents the set of price-quantity quadruples that could occur given that manager i responds optimally to the incentives he is given. Throughout this section, s, t, x, y will stand for elements of the set $\{p, q\}$ and will be used to denote either price or quantity competition in various contexts.[11] Let $q_i(p_1, p_2)$ be the demand function for product i, $\Theta_i^s \subseteq \Re^k$ (where k is a positive integer) be the set of incentive parameters available to owner i when her firm competes by setting s, and θ_i^s be a generic element of Θ_i^s. Let $U_i\left(p_1, p_2, q_1, q_2 \mid \theta_i^s, t_j\right)$ be manager i's utility function conditional on owner i's incentive parameter choice, θ_i^s, and strategy choice t_j by manager j. If firm i competes by setting $s \in \{p, q\}$ and firm j competes by setting $t \in \{p, q\}$, denote the outcome set for player i by

$$\Omega_i^{st}\left(\theta_i^s\right) = \left\{ \begin{array}{c} \left(p_1, p_2, q_1, q_2\right) : q_1 = q_1\left(p_1, p_2\right), q_2 = q_2\left(p_1, p_2\right), \text{ and} \\ s_i \in \arg\max U_i\left(p_1, p_2, q_1, q_2 \mid \theta_i^s, t_j\right) \end{array} \right\}.$$

Hence the outcome set represents the projection of manager i's best response function into the four-dimensional price-quantity space.[12] The set of second-stage equilibrium outcomes when incentive parameters θ_i^s and θ_j^t are chosen is given by the intersection of the two sets, $\Omega_i^{st}(\theta_i^s) \cap \Omega_j^{ts}(\theta_j^t)$.

Now consider the first-stage equilibrium. The owner chooses incentive parameters in order to maximize profit subject to the constraint that the resulting prices and quantities represent a second-stage equilibrium outcome, given the choice of incentive parameters by the other owner. Assuming firm i sets $s \in \{p, q\}$ and j sets $t \in \{p, q\}$ in the second stage, in the first stage the owner solves

$$\max_{\theta_i^s \in \Theta_i^s} \left(p_i - c_i\right) q_i \tag{13}$$

$$\text{subject to} \qquad \left(p_1, p_2, q_1, q_2\right) \in \Omega_i^{st}\left(\theta_i^s\right) \cap \Omega_j^{ts}\left(\bar{\theta}_j^t\right),$$

[11] We will use s_i to denote the particular strategy choice by manager i when he competes by setting s. For example, when $s = p$, $s_i = p_i$ stands for the particular price he chooses.

[12] Although the result is robust to allowing the manager's best response mapping to be multivalued, for expositional ease we will restrict the discussion to situations where the manager has a unique best response.

where $\bar{\theta}_j^t$ is firm j's equilibrium incentive parameter choice. When the firm's problem is stated as in (13), it is clear that what the owner really does when she chooses her incentive parameter is choose the point out of her rival's outcome set that maximizes her firm's profit, subject to the constraint that for some incentive parameter that point is also in her own outcome set. Thus, holding the rival's outcome set fixed, if for either firm any outcome set that can be generated when it chooses s and its opponent chooses t can also be generated when it chooses x and its opponent chooses y (by possibly different incentive parameters), then the equivalence result follows immediately. Formally, this condition can be stated as:

Outcome Set Equivalence (OSE). For player $i \in \{1, 2\}$, for any $s, t, x, y \in \{p, q\}$, and any $\theta_i^s \in \Theta_i^s$, there exists a $\theta_i^x \in \Theta_i^x$ such that $\Omega_i^{xy}(\theta_i^x) = \Omega_i^{st}\left(\theta_i^s\right)$.

If condition OSE holds, the distinction between price, quantity, and mixed competition reduces to mere differences in the naming of outcome sets. While sufficient for the equivalence result, condition OSE is stronger than necessary. The following two assumptions provide a weaker sufficient condition:

Replication. For any $s, t, x, y \in \{p, q\}$, there exists a $\hat{\theta}_i^x \in \Theta_i^x$ such that $\Omega_i^{st}(\bar{\theta}_i^s) = \Omega_i^{xy}(\hat{\theta}_i^x)$.

Feasibility. For any $(p_1, p_2, q_1, q_2) \in \Omega_j^{ts}(\bar{\theta}_j^t)$ there exists a $\theta_i^s \in \Theta_i^s$ such that $(p_1, p_2, q_1, q_2) \in \Omega_i^{st}(\theta_i^s)$ if and only if there exists a $\theta_i^x \in \Theta_i^x$ such that $(p_1, p_2, q_1, q_2) \in \Omega_i^{xy}(\theta_i^x)$.

Replication requires that the equilibrium outcome set when the firms compete by setting s and t can be exactly duplicated when the firms compete by setting x and y. *Replication* is a strong requirement, but it applies only to the equilibrium outcome sets and therefore is a significant weakening of *OSE*. *Feasibility* requires that any point in the other player's equilibrium outcome set that lies in some outcome set for firm i when the firms compete by setting s and t also lies in some outcome set when the firms compete by setting x and y. Thus it is a requirement that the set of feasible outcomes in firm i's first-stage optimization problem be independent of both whether it chooses price or quantity and whether the other firm chooses price or quantity.

Note that with either the replication or feasibility conditions, if $t = y$ the condition is trivially satisfied. As Klemperer and Meyer (1986) note in nondelegation games, holding the strategic choice of one's rival fixed, a firm is indifferent between choosing price and choosing quantity. The same is true in our delegation context. Fixing the choice of incentive parameters by both firms, any outcome set that a firm can achieve when its manager sets price will also be achieved when its manager sets quantity by the same choice of incentive parameters. Thus the true demands of the conditions are on replication and feasibility with regard to changes in one's opponent's strategic variable.

With these conditions in place, we state the general form of the equivalence result.

Proposition 3. If the replication and feasibility conditions hold, then the subgame-perfect equilibrium prices and quantities are the same regardless of whether the firms compete in prices, the firms compete in quantities, or one firm chooses price and the other chooses quantity.

Proof. Suppose that firm i sets s and firm j sets t in the second-stage competition. We will show that the same prices and quantities are an equilibrium outcome when firm i sets x and firm j sets y. Firm i's profit-maximization problem is

$$\max_{\theta_i^s \in \Theta_i^s} (p_i - c_i) q_i \tag{14}$$
$$\text{subject to} \qquad (p_1, p_2, q_1, q_2) \in \Omega_i^{st}\left(\theta_i^s\right) \cap \Omega_j^{ts}\left(\bar{\theta}_j^t\right).$$

The firm does not directly care about the incentive parameters; only the prices and quantities are payoff-relevant. The set of feasible prices and quantities is given by

$$\left\{(p_1, p_2, q_1, q_2) : (p_1, p_2, q_1, q_2) \in \Omega_i^{st}\left(\theta_i^s\right) \cap \Omega_j^{ts}\left(\bar{\theta}_j^t\right) \text{ for some } \theta_i^s \in \Theta_i^s\right\}.$$

But by replication there exists a $\hat{\theta}_j^y \in \Theta_j^y$ such that $\Omega_j^{ts}\left(\bar{\theta}_j^t\right) = \Omega_j^{yx}\left(\hat{\theta}_j^y\right)$, and so this set is identical to

$$\left\{(p_1, p_2, q_1, q_2) : (p_1, p_2, q_1, q_2) \in \Omega_i^{st}\left(\theta_i^s\right) \cap \Omega_j^{yx}\left(\hat{\theta}_j^y\right) \text{ for some } \theta_i^s \in \Theta_i^s\right\}.$$

By feasibility, this set is identical to

$$\left\{(p_1, p_2, q_1, q_2) : (p_1, p_2, q_1, q_2) \in \Omega_i^{xy}\left(\theta_i^x\right) \cap \Omega_j^{yx}\left(\hat{\theta}_j^y\right) \text{ for some } \theta_i^x \in \Theta_i^x\right\}.$$

Since the feasible set in

$$\max_{\theta_i^x \in \Theta_i^x} (p_i - c_i) q_i$$

$$\text{subject to} \qquad (p_1, p_2, q_1, q_2) \in \Omega_i^{xy}\left(\theta_i^x\right) \cap \Omega_j^{yx}\left(\hat{\theta}_j^y\right)$$

is the same as in (14) and only prices and quantities are payoff-relevant, firm i must choose an incentive parameter that results in the same prices and quantities as in (14), provided that one exists. By replication, there exists a $\hat{\theta}_i^x \in \Theta_i^x$ such that $\Omega_i^{st}(\bar{\theta}_i^s) = \Omega_i^{xy}(\hat{\theta}_i^x)$, which implies the same prices and quantities, and so $\hat{\theta}_i^x$ is a best response to $\hat{\theta}_j^y$. Reversing the roles of i and j completes the proof. *Q.E.D.*

For the intuition behind the proof, consider comparing the equilibrium outcomes when both firms set prices and when both firms set quantities. Owner i chooses her incentive parameter to select a point off of firm j's equilibrium reaction function that maximizes her profit. If any point that she can select when the firms set quantities can also be selected when the firms set prices, then the feasible sets in the two versions of the problem are the same. Since the feasible sets are the same, firm i must choose the same price-quantity point in either case. Although there may be many incentive parameters that select this point, choosing the one that replicates the equilibrium outcome set in the price-setting game does so, and it ensures that if owner j also chooses to replicate her equilibrium outcome set from the price-setting game, the firms' choices of incentive parameters in the quantity-setting game comprise a subgame-perfect Nash equilibrium.

Remark 1. A weaker version of the replication condition requiring only replication near the equilibrium outcome is generally not sufficient for the result, although it is sufficient in the linear model, where local replication is equivalent to global replication. The reason is that the local replication of the equilibrium outcome set may include higher-profit points that were not in the original equilibrium outcome set.

Remark 2. Proposition 3 is easily extended to the case where each type of delegation game (price-price, price-quantity, quantity-quantity) has multiple equilibria. In this environment the equivalence result states that for any equilibrium in one type of competition, if the feasibility and Replication conditions hold for that equilibrium, then there is an equilibrium in either other type of competition that achieves the same prices and quantities.

Remark 3. Although the general proof is not subject to any of the restrictions of the linear model, in more general environments the equilibria quickly become very difficult to compute parametrically. In addition, the more complicated the game gets, the more difficult it is to determine which values of the incentive parameters can be replicated, and questions of existence and uniqueness of the equilibrium become more important. As the number of incentive parameters grows, the game begins to resemble competition in supply functions, as in Grossman (1981) and Klemperer and Meyer (1989). The danger here is not so much that the equivalence result will not hold, but that there will be a great multitude of equilibrium outcomes, each of which can be supported by incentives in either price-setting, quantity-setting, or mixed games. We illustrate this point and make the connection between our game and linear supply function equilibria in the next subsection.

□ **Application to the linear model.** The linear-demand, relative-performance case provides a natural illustration of the logic of the general proof. Refer once again to Figure 1. Direct computation in the linear model reveals that when the firm competes in prices, in the equilibrium of the delegation game the owner sets the incentive parameters so that the managers' reaction functions are EG for manager 1 and FH for manager 2. Taking this as given, it is straightforward to verify the conditions of Proposition 3 for the case where managers compete in quantities. To verify feasibility, note that in the quantity-setting game varying the weight on relative performance pivots manager 1's reaction function through point E and manager 2's reaction function through point F. Thus any point along FH can be achieved through some choice of incentive parameters by owner 1, and similarly any point along EG can be achieved by owner 2. And since reaction functions are linear and all quantity reaction functions contain point E (respectively F), there is a reaction function connecting points E and G (respectively F and H) in the quantity-setting game. This verifies the replication condition.[13] Since the two conditions are satisfied, by Proposition 3 point I is the equilibrium outcome in both price-price and quantity-quantity competition. A similar argument shows that point I is also the equilibrium in price-quantity competition.

Proposition 3 allows us to generalize the equivalence result in the linear case beyond relative performance incentive schemes. Replication and feasibility are requirements that the owners have sufficient control over the behavior of their respective managers. In environments where they do not have enough control, the equivalence result will fail to hold. For example, in the model employed by Fershtman and Judd (1987), where owners compensate managers based on a linear combination of profits and sales, varying the incentive parameter shifts the manager's reaction curve in a parallel manner but cannot change its slope. Consequently there is no outcome set that can be the outcome of delegation in both the price-setting and quantity-setting games, and the equivalence result does not hold.

If we restrict ourselves to incentive schemes that result in linear reaction functions for the managers, schemes that involve unrestricted, independent control of the function's slope and intercept are the most general class. We will call games with linear demand and constant marginal cost where owners have this degree of control *linear, unrestricted, independent-control delegation games*, or LUIC. Independent control of the slope and intercept can be achieved with as few as two incentive parameters for each firm. For example, setting independent weights on own revenue and rival firm's revenue gives independent control of the slope and intercept. However, it is possible to specify two incentive parameters that do not imply independent control of slope and intercept, such as setting weight on own cost and rival's cost.

An important example of LUIC delegation games is where firms compete by choosing linear supply functions. As studied by Grossman (1981) and Klemperer and Meyer (1989), a supply-function strategy is a function $\sigma_i(p)$ that specifies the quantity of a good a firm produces as a function of its price. In the absence of uncertainty, supply function competition generally results in multiple equilibria. For example, Klemperer and Meyer (1989) show that in homogeneous-products oligopolies, any nonnegative price-quantity vector is an equilibrium outcome when firms compete by setting supply functions.

When firms compete by setting linear supply functions, the resulting game is equivalent to a LUIC delegation game. A linear supply function strategy is any supply function strategy that can be written as $\sigma_i(p_i) = \lambda_i + \mu_i p_i$. Combining the demand function (10) with the linear supply function strategy yields

$$q_i = a_i - b_i p_i + z_i p_j = \lambda_i + \mu_i p_i,$$

[13] Note that in the linear model, OSE does not hold. The fact that the only reaction function that is replicable in this model is the equilibrium reaction function illustrates the need for the weaker requirement of replication and feasibility. A justification for why the only replicable reaction function should be the equilibrium one in the linear case is provided by the geometric analysis in Section 3.

or

$$p_i = \frac{a_i - \lambda_i + z_i p_j}{b_i + \mu_i}.$$

Hence by choosing λ_i and μ_i, owners exercise unrestricted, independent control of the reaction curve's slope and intercept.

Corollary 3 extends the equivalence result to LUIC delegation games, including games where firms set linear supply functions.

Corollary 3. In LUIC delegation games, the set of equilibrium price-quantity quadruples is the same, regardless of whether the firms compete by setting prices, quantities, linear supply functions, or any combination of these.

Proof. Note that condition OSE is satisfied. *Q.E.D.*

The preceding argument shows that the sets of equilibrium outcomes under LUIC delegation and supply-function competition are the same. We now extend the Klemperer and Meyer (1989) result that any outcome can be an equilibrium to the linear model with LUIC delegation or linear supply-function competition.

Proposition 4. In an LUIC delegation game, any strictly positive price-quantity quadruple satisfying the demand system is an equilibrium of the two-stage game, regardless of whether the firms compete by setting prices, quantities, linear supply functions, or any combination of these.

Proof. We provide a sketch of the proof. The details are available from the authors upon request. Consider a nonnegative price-quantity quadruple, $(p_1^*, p_2^*, q_1^*, q_2^*)$, satisfying the demand system (10). Let $\pi_i(q_1, q_2)$ be firm i's profit as a function of the quantities. In the quantity space, consider the profit isoquant for firm i through point (q_1^*, q_2^*), $\pi_i(q_1, q_2) = \pi_i(q_1^*, q_2^*)$, and suppose that firm j chooses as its linear reaction function the line that is tangent to this isoquant at (q_1^*, q_2^*). Call this line $r_j^*(q_i)$. Owner i's strategy choice (regardless of the type of competition) amounts to choosing the point off of $r_j^*(q_i)$ that maximizes her profit. But, by construction, this point is (q_1^*, q_2^*). Hence any linear best response to $r_j^*(q_i)$ will intersect $r_j^*(q_i)$ at (q_1^*, q_2^*). One such curve is the line through (q_1^*, q_2^*) that is tangent to firm j's profit isoquant. Call this line $r_i^*(q_j)$. Repeating the argument shows that $r_j^*(q_i)$ is a best response to $r_i^*(q_j)$, and hence that the equilibrium outcome is (q_1^*, q_2^*). *Q.E.D.*

Corollary 3 and Proposition 4 provide an illustration of how the equivalence result extends to environments with multiple equilibria. For each nonnegative quantity vector (q_1^*, q_2^*) there is an equilibrium that supports it in delegated price competition, delegated quantity competition, competition in linear supply functions, or any combination of these. Of course, while the outcome can be supported by any type of strategy, the exact incentive parameters (or supply function parameters) the firm must choose to do so will depend on the strategy employed by its rival, just as in the case of a unique equilibrium.

While any point can be an equilibrium, the incentive parameters needed to support a particular equilibrium may not always be reasonable. For example, it may be that the supply-function strategy needed to support a particular outcome is downward sloping. Prohibiting the firm from employing such strategies will shrink the set of equilibrium outcomes.[14] However, as long as these restrictions are stated in behavioral terms, e.g., the relationship between q_i and p_i implied by the firm's incentive scheme cannot be downward sloping, all types of second-stage competition will be restricted in the same way, and the equivalence result will continue to hold. On the other hand, if the restriction placed on the set of feasible incentive schemes is technical, e.g., the firm cannot place positive weight on the other firm's profit in its manager's incentive scheme, the

[14] Vives (1999) observes that a delegation game such as Fershtman and Judd's (1987) can be interpreted as placing restrictions on the set of feasible supply functions, the result of the restriction being that the set of equilibria of the game shrinks.

equivalence result will not necessarily follow. Thus our result in the linear, relative-performance case (Propositions 1 and 2) can be interpreted as stating a restriction on the type of managerial behavior that can be induced by the owner, namely allowing only behavior that can be generated by optimizing a linear combination of the firm's profit and its rival's profit. The result is that the set of equilibria shrinks to a single point, but the equivalence result continues to hold.

5. Conclusion

■ If owners have sufficient control over the behavior of their managers, the differences between the outcomes of price and quantity competition dissolve once embedded in a delegation game. This suggests that if one's goal is to understand the behavior of a firm competing in an oligopoly, understanding its internal organization, i.e., manager's incentives, is at least as important as understanding whether its strategic variable in the product market is price, quantity, or some other instrument such as a supply function.

A number of articles have considered the question of whether, given the choice, firms should compete in prices or quantities. Singh and Vives (1984) show that it is a dominant strategy for firms to set quantities (prices) when the products are substitutes (complements). Klemperer and Meyer (1986) study differentiated-products duopoly environments and look for Nash equilibria of the game where the firms' strategic choice involves choosing whether to set prices or quantities. They show that in the absence of uncertainty there are three types of equilibria (both firms set price, both firms set quantity, and one sets price and the other sets quantity), each with a different outcome. The present article's contribution on this subject is to point out that as owners use their ability to manipulate incentives to gain a strategic advantage in the two-stage game, the gap between the manager's behavior under price and quantity competition narrows and even disappears. Ultimately, the equilibrium requirements have to do with the behavior of the managers, and it does not matter whether this behavior arises from delegated price competition, quantity competition, or even supply-function competition. Thus in trying to understand the behavior of competing firms, the firm's choice of product-market strategic variable is only part of the story. A full understanding of the competitive environment requires an understanding of the manager's product-market behavior, which results from the combination of the nature of product-market competition and the internal organization of the firm.

The results presented in this article are robust to extension. In particular, they are valid for any finite number of firms. They are also valid for more general demand structures, subject to the provisos related to computability, difficulty in satisfying the conditions for the theorem, and existence and uniqueness of equilibria mentioned earlier. However, the result need not be limited to providing an invariance result in the case of price competition and quantity competition. The same reasoning should apply to any game where the players' payoffs depend on two different sets of variables (e.g., prices and quantities) but specifying one set of variables determines the other. If the owners have sufficient control over the behavior of their managers, then the outcome of the delegation game should not depend on the type of second-stage competition.

Appendix

■ In this Appendix we derive the equivalence result in the linear demand, constant marginal cost case via direct computation, providing proofs for the claims in Propositions 1 and 2 and Corollaries 1 and 2.[15] The first three subsections derive the equilibrium incentive parameters, prices, and quantities for the cases in which both firms choose quantities, both firms choose prices, and firm 1 chooses price and firm 2 chooses quantity. The fourth subsection compares the equilibrium prices, quantities, and profits in the delegation game with the corresponding values in nondelegation price and quantity competition. A brief discussion of the results is also provided.

[15] For ease of presentation, some steps representing standard operations are mentioned but the corresponding algebra is omitted. A full presentation containing all steps is available from the authors upon request.

□ **Both firms choose quantity.** Let θ_i^q be the weight the owner of firm i puts on the profit of firm j in manager i's incentive scheme when both firms compete in quantity. Manager i has the objective function

$$\left(p_i\left(q_i, q_j\right) - c_i\right) q_i + \theta_i^q \left(p_j\left(q_i, q_j\right) - c_i\right) q_j .$$

Differentiating with respect to q_i, setting the result equal to zero, and solving for q_i yields the manager's reaction function.[16] Solving the reaction functions for q_1 and q_2 gives second-stage equilibrium quantities:

$$q_i\left(\theta_i^q, \theta_j^q\right) = \frac{2\beta_j\left(\alpha_i - c_i\right) + \left(c_j - \alpha_j\right)\left(\gamma_i + \theta_i^q \gamma_j\right)}{4\beta_i\beta_j - \left(\theta_i^q \gamma_j + \gamma_i\right)\left(\theta_j^q \gamma_i + \gamma_j\right)}.$$

In the first stage, each owner maximizes profit by choosing the incentive parameter θ_i^q, subject to the constraint that the managers play an equilibrium in the second stage. Hence the owner's objective function is given by

$$x_i^q\left(\theta_i^q\right) = \left[p_i\left(q_i\left(\theta_i^q, \theta_j^q\right), q_j\left(\theta_i^q, \theta_j^q\right)\right) - c_i\right] q_i\left(\theta_i^q, \theta_j^q\right).$$

Differentiating with respect to θ_i^q and setting the result equal to zero yields the owner's reaction function. Solving the two reaction functions yields the equilibrium value of the incentive parameter:[17]

$$\bar{\theta}_i^q = \frac{-\gamma_i\left(\left(\alpha_i - c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right)}{\beta_j\left(\left(\alpha_j - c_j\right)\beta_i - \left(\alpha_i - c_i\right)\gamma_j\right) + \left(\alpha_j - c_j\right)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}. \tag{A1}$$

Note that the sign of $\bar{\theta}_i^q$ is opposite the sign of γ_i. The values in (A1) imply equilibrium prices and quantities:

$$\bar{q}_i^q = \frac{1}{4}\left(\frac{\beta_i\left(\left(\alpha_i - c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right) + \left(\alpha_i - c_i\right)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}{\beta_i\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}\right) \tag{A2}$$

$$\bar{p}_i^q = \frac{2\beta_j\left(\alpha_i - c_i\right) - \left(\alpha_j - c_j\right)\gamma_i}{4\beta_j} + c_i. \tag{A3}$$

Under our assumptions on the demand system in (11) and (12), $\bar{q}_i^q > 0$, and $\bar{p}_i^q > c$. These assumptions are appealing in that they have a natural economic interpretation. However, they are not necessary for the delegation game's equilibrium to involve positive quantities and profits. The necessary and sufficient condition for this is found by requiring the numerator of (A2) and the numerator of the first term of (A3) to be nonnegative. However, these conditions lack a ready economic interpretation.

□ **Both firms choose price.** Let θ_i^p be the weight the owner of firm i assigns to the profit of firm j when the managers compete by setting prices in the second stage. In this case, the manager's objective function is given by

$$\left(p_i - c_i\right) q_i\left(p_i, p_j\right) + \theta_i^p\left(p_j - c_j\right) q_j\left(p_j, p_i\right).$$

Differentiating this expression with respect to p_i, setting the result equal to zero, and solving for p_i yields manager i's reaction curve in the second-stage game as a function of the incentive parameter θ_i^p. Solving the reaction functions yields the second-stage equilibrium prices:

$$p_i\left(\theta_i^p, \theta_j^p\right) = \frac{2a_ib_i + 2b_ib_jc_i + z_i\left(\left(a_j + b_jc_j\right) - \theta_j^p c_iz_i\right) + \theta_i^p\left(a_j - b_jc_j - c_i\theta_j^p z_i\right)z_j}{4b_ib_j - \left(\theta_j^p z_i + z_j\right)\left(z_i + \theta_i^p z_j\right)}. \tag{A4}$$

Given the second-stage equilibrium, the owners choose θ_i^p in order to maximize

$$\left(p_i\left(\theta_i^p, \theta_j^p\right) - c_i\right) q_i\left(p_i\left(\theta_i^p, \theta_j^p\right), p_j\left(\theta_i^p, \theta_j^p\right)\right).$$

[16] The second derivative of the manager's objective function with respect to q_i is $-2\beta_i$. Hence the second-order conditions are satisfied on the reaction function.

[17] Given the assumptions we have made on the demand system,the second-order conditions can be verified at the equilibrium values of the incentive parameters.

Differentiating with respect to θ_i^p, setting the result equal to zero, and solving for θ_i^p and θ_j^p yields equilibrium values for the incentive parameters (after substituting in for a_i, b_i, z_i in terms of $\alpha_i, \beta_i, \gamma_i$)

$$\bar{\theta}_i^p = \frac{(\alpha_i - c_i)\gamma_i}{2\beta_i\left(\alpha_j - c_j\right) - (\alpha_i - c_i)\gamma_j}.$$

Substituting these values into (A4) yields the same quantities and prices as (A4) and (A3).

□ **One firm chooses price and one firm chooses quantity.** Let ϕ_i be the weight that firm i puts on the profit of its rival. In the second stage, the manager of firm 1 chooses p_1 to maximize

$$(p_1 - c_1)\tilde{q}_1(p_1, q_2) + \phi_1(\tilde{p}_2(p_1, q_2) - c_2)q_2$$

and the manager of firm 2 chooses q_2 to maximize

$$(\tilde{p}_2(p_1, q_2) - c_2)q_2 + \phi_2(p_1 - c_1)\tilde{q}_1(p_1, q_2),$$

where

$$\tilde{q}_1(p_1, q_2) = \frac{\alpha_1 - p_1 + \gamma_1 q_2}{\beta_1},$$

and

$$\tilde{p}_2(p_1, q_2) = \frac{(\alpha_2\beta_1 - \alpha_1\gamma_2) + p_1\gamma_2 - q_2(\beta_1\beta_2 - \gamma_1\gamma_2)}{\beta_1}.$$

Differentiating the objective functions with respect to their respective control variables, setting the results equal to zero, and solving for p_1 and q_2 yields second-stage equilibrium

$$p_1(\phi_1, \phi_2) = \frac{(\gamma_1 - \phi_1\gamma_2)(c_2\beta_1 - \alpha_2\beta_1 - c_1\phi_2\gamma_1 + \alpha_1\gamma_2) + 2(c_1 + \alpha_1)(\beta_1\beta_2 - \gamma_1\gamma_2)}{4\beta_1\beta_2 - \phi_2\gamma_1^2 - 3\gamma_1\gamma_2 + \phi_1\phi_2\gamma_1\gamma_2 - \phi_1\gamma_2^2} \tag{A5}$$

$$q_2(\phi_1, \phi_2) = \frac{2(\alpha_2 - c2)\beta_1 + (c_1 - \alpha_1)(\phi_2\gamma_1 + \gamma_2)}{4\beta_1\beta_2 - \phi_2\gamma_1^2 - 3\gamma_1\gamma_2 + \phi_1\phi_2\gamma_1\gamma_2 - \phi_1\gamma_2^2}. \tag{A6}$$

In the first stage the owners of the firms 1 and 2 choose ϕ_1 and ϕ_2 to maximize

$$(p_1(\phi_1, \phi_2) - c_1)\tilde{q}_1(p_1(\phi_1, \phi_2), q_2(\phi_1, \phi_2))$$

and

$$(\tilde{p}_2(p_1(\phi_1, \phi_2), q_2(\phi_1, \phi_2)) - c_2)q_2(\phi_1, \phi_2)$$

respectively. Differentiating with respect to the control variables, setting the result equal to zero, and solving for ϕ_1 and ϕ_2 yields equilibrium incentive parameters:

$$\bar{\phi}_1 = \frac{-\gamma_1((\alpha_1 - c_1)\beta_2 - (\alpha_2 - c_2)\gamma_1)}{\beta_2((\alpha_2 - c_2)\beta_1 - (\alpha_1 - c_1)\gamma_2) + (\alpha_2 - c_2)(\beta_2\beta_1 - \gamma_2\gamma_1)}$$

$$\bar{\phi}_2 = \frac{(\alpha_2 - c_2)\gamma_2}{2\beta_2(\alpha_1 - c_1) - (\alpha_2 - c_2)\gamma_1}.$$

It follows from the assumptions made on the demand functions that $\bar{\phi}_1$ has the opposite sign of γ_1 and $\bar{\phi}_2$ has the same sign as γ_2. Substituting these values into (A5) and (A6) yields the same quantities and prices as (A4) and (A3).

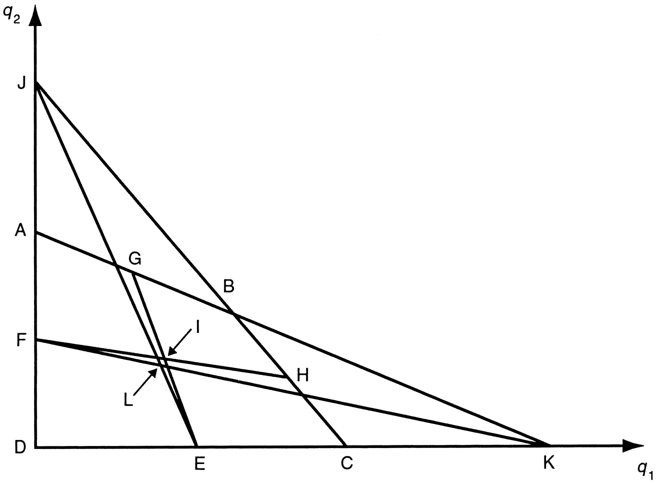
□ **Comparison with nondelegation games.** For reference, the equilibria in nondelegation competition in this market are given by

Quantity competition

$$p_i^{qnd} = \frac{\beta_i\left(2(\alpha_i + c_i)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right) - c_i\gamma_i\gamma_j}{4\beta_i\beta_j - \gamma_i\gamma_j}$$

$$q_i^{qnd} = \frac{2(\alpha_i - c_i)\beta_j - (\alpha_j - c_j)\gamma_i}{4\beta_i\beta_j - \gamma_i\gamma_j}$$

FIGURE A1



Price competition

$$p_i^{pnd} = \frac{\beta_i \left(2\left(\alpha_i + c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i - \alpha_i\gamma_i\gamma_j\right)}{4\beta_i\beta_j - \gamma_i\gamma_j}$$

$$q_i^{pnd} = \beta_j\left(\frac{\beta_i\left(\left(\alpha_i - c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right) + \left(\alpha_i - c_i\right)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)}{\left(\beta_i\beta_j - \gamma_i\gamma_j\right)4\beta_i\beta_j - \gamma_i\gamma_j}\right),$$

where the superscripts *qnd* and *pnd* denote nondelegation quantity and price competition respectively. Throughout this subsection, we denote equilibrium values of the delegation game with a bar. Substituting in the equilibrium prices yields

$$p_i^{qnd} - \bar{p}_i = \frac{\gamma_i\gamma_j\left(2\left(\alpha_i - c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right)}{4\beta_j\left(4\beta_i\beta_j - \gamma_i\gamma_j\right)} \tag{A7}$$

$$\bar{p}_i - p_i^{pnd} = \frac{\gamma_i\gamma_j\left(2\left(\alpha_i - c_i\right)\beta_j + \left(\alpha_j - c_j\right)\gamma_i\right)}{4\beta_j\left(4\beta_i\beta_j - \gamma_i\gamma_j\right)}. \tag{A8}$$

The right-hand side of (A7) is positive, since $(\alpha_i - c_i)\beta_j - (\alpha_j - c_j)\gamma_i > 0$. Setting the right-hand side of (A8) greater than zero and rearranging yields the condition in Corollary 1(ii).

Substituting in the equilibrium quantities yields

$$q_i^{pnd} - \bar{q}_i = \frac{\gamma_i\gamma_j\left(\beta_i\left(\left(\alpha_i - c_i\right)\beta_j - \left(\alpha_j - c_j\right)\gamma_i\right) + \left(\alpha_i - c_i\right)\left(\beta_i\beta_j - \gamma_i\gamma_j\right)\right)}{4\beta_i\left(\beta_i\beta_j - \gamma_i\gamma_j\right)\left(4\beta_i\beta_j - \gamma_i\gamma_j\right)} \tag{A9}$$

$$\bar{q}_i - q_i^{qnd} = \frac{\gamma_i\gamma_j\left(\left(\alpha_i - c_i\right)\left(2\beta_i\beta_j + \gamma_i\gamma_j\right) - 3\left(\alpha_j - c_j\right)\beta_i\gamma_i\right)}{4\beta_i\left(\beta_i\beta_j - \gamma_i\gamma_j\right)\left(4\beta_i\beta_j - \gamma_i\gamma_j\right)}. \tag{A10}$$

The right-hand side of (A9) is positive, since $(\alpha_i - c_i)\beta_j - (\alpha_j - c_j)\gamma_i > 0$ and $\beta_i\beta_j - \gamma_i\gamma_j > 0$. Rearranging the right-hand side of (A10) yields the condition in Corollary 2(ii).

The conditions in Corollaries 1(ii) and 2(ii) are best interpreted geometrically in terms of Figure A1, which is a slight variation on Figure 1.[18]

Points A–I represent the same points as in Figure 1, and points J and K represent the extension of lines AB and BC to the axes. Thus lines AK and CJ are the translations of the lines $p_2 = c_2$ and $p_1 = c_1$ to the quantity space. Lines FK and EJ respectively represent the reaction curves for firms 1 and 2 in nondelegation quantity competition, and thus point L is the nondelegation quantity competition equilibrium.[19] Point I remains the equilibrium point in the delegation game.

The condition in Corollary 2(ii) amounts to saying that $\bar{q}_2 > q_2^q$ if and only if the midpoint of segment GK is to the right of point B. This will tend to fail if point C is close to point K.[20] As point C approaches point K, point H (the mid-

[18] The coordinates of each of the points in the diagram are given in Table A1.

[19] In nondelegation price reaction functions, connect points J and G for manager 1 and K and H for manager 2.

[20] At point E, $q_1 = (a_1 - c_1)/\beta_1$. At point K, $q_1 = (a_2 - c_2)/\gamma_2$. Hence the conditions in Corollaries 2(ii) and 1(ii) are closely related, except for the fact that 2(ii) can be violated only when the goods are substitutes and 1(ii) can be violated only when the goods are complements.

TABLE A1 **Value of Outcome Variable**

Point	q_1	q_2	p_1	p_2
A	0	$\frac{\alpha_2-c_2}{\beta_2}$	$\frac{\alpha_1\beta_2-(\alpha_2-c_2)\gamma_1}{\beta_2}$	c_2
B	$\frac{(\alpha_1-c_1)\beta_2-(\alpha_2-c_2)\gamma_1}{\beta_1\beta_2-\gamma_1\gamma_2}$	$\frac{(\alpha_2-c_2)\beta_1-(\alpha_1-c_1)\gamma_2}{\beta_1\beta_2-\gamma_1\gamma_2}$	c_1	c_2
C	$\frac{\alpha_1-c_1}{\beta_1}$	0	c_1	$\frac{\alpha_2\beta_1-(\alpha_1-c_1)\gamma_2}{\gamma_1}$
D	0	0	α_1	α_2
E	$\frac{\alpha_1-c_1}{2\beta_1}$	0	$\frac{\alpha_1+c_1}{2}$	$\frac{2\alpha_2\beta_1-(\alpha_1-c_1)\gamma_2}{2\beta_1}$
F	0	$\frac{\alpha_2-c_2}{2\beta_2}$	$\frac{2\alpha_1\beta_2-(\alpha_2-c_2)\gamma_1}{2\beta_2}$	$\frac{\alpha_2+c_2}{2}$
G	$\frac{(\alpha_1-c_1)\beta_2-(\alpha_2-c_2)\gamma_1}{2(\beta_1\beta_2-\gamma_1\gamma_2)}$	*	$\frac{(\alpha_1+c_1)\beta_2-(\alpha_2-c_2)\gamma_1}{2\beta_2}$	c_2
H	**	$\frac{(\alpha_2-c_2)\beta_1-(\alpha_1-c_1)\gamma_2}{2(\beta_1\beta_2-\gamma_1\gamma_2)}$	c_1	$\frac{(\alpha_2+c_2)\beta_1-(\alpha_2-c_2)\gamma_2}{2\beta_1}$
I	#	#	#	#
J	0	$\frac{\alpha_1-c_1}{\gamma_1}$	c_1	$\frac{\alpha_2\gamma_1-(\alpha_1-c_1)\beta_2}{\gamma_1}$
K	$\frac{\alpha_2-c_2}{\gamma_2}$	0	$\frac{\alpha_1\gamma_2-(\alpha_2-c_2)\beta_1}{\gamma_2}$	c_2
L	##	##	##	##

$$* = \frac{\beta_2((\alpha_2-c_2)\beta_1-(\alpha_1-c_1)\gamma_2)+(\alpha_2-c_2)(\beta_1\beta_2-\gamma_1\gamma_2)}{2\beta_2(\beta_1\beta_2-\gamma_1\gamma_2)}$$

$$** = \frac{\beta_1((\alpha_1-c_1)\beta_2-(\alpha_2-c_2)\gamma_1)+(\alpha_1-c_1)(\beta_1\beta_2-\gamma_1\gamma_2)}{2\beta_1(\beta_1\beta_2-\gamma_1\gamma_2)}$$

$$\# : \bar{q}_i = \frac{1}{4}\left(\frac{\beta_i((\alpha_i-c_i)\beta_j-(\alpha_j-c_j)\gamma_i)+(\alpha_i-c_i)(\beta_i\beta_j-\gamma_i\gamma_j)}{\beta_i(\beta_i\beta_j-\gamma_i\gamma_j)}\right), \text{ and } \bar{p}_i = \frac{2(\alpha_i+c_i)\beta_j-(\alpha_j-c_j)\gamma_i}{4\beta_j}.$$

$$\#\# : p_i^{qnd} = \frac{\beta_i(2(\alpha_i+c_i)\beta_j-(\alpha_j-c_j)\gamma_i)-c_i\gamma_i\gamma_j}{4\beta_i\beta_j-\gamma_i\gamma_j}, \text{ and } q_i^{qnd} = \frac{2(\alpha_i-c_i)\beta_j-(\alpha_j-c_j)\gamma_i}{4\beta_i\beta_j-\gamma_i\gamma_j}.$$

point of BC) moves toward line FK, which means that the delegation-game reaction function for firm 2 approaches its quantity-setting reaction function. At the same time, point G (the midpoint of AB) moves to the right, increasing the slope of EG, firm 1's delegation reaction function. The result is that the delegation equilibrium quantity, point I, moves down EG toward FK. Since FK is downward sloping, for C close enough to K the delegation equilibrium point I has a smaller q_2 coordinate than the quantity-setting equilibrium point, L.

The condition in Corollary 1(ii) amounts to the requirement that distance DE be less than distance DK. Geometrically, the significance of this condition is the same when the problem is stated in the (p_1, p_2) space. Note that DE will always be less than DK when the goods are substitutes. However, when the goods are complements, point K has $q_1 < 0$, in which case it is possible for the length of DK to be less than DE.

Table A1 provides coordinates for each of the points in Figures 1 and A1.

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