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Relative Performance as a Strategic Commitment Mechanism

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Can managers' personality traits be of use to profit maximizing firm owners? We investigate the case where managers have a variety of attitudes toward relative performance that are indexed by their type. We consider two stage games where profit maximizing owners select managers in the first stage, and these managers, knowing each other's types, compete in a duopoly game in the second stage. The equilibria of various types of competition are derived and comparisons are made to the standard case where managers are profit maximizers. We show that managers' types can be used as a strategic commitment device that can increase firm profits in certain environments. Copyright © 2002 John Wiley & Sons, Ltd.

INTRODUCTION

Traditional economic and industrial organization theories tend to view firms as entities whose sole objective is to maximize their own profits. Recent researchers have criticized this view as an oversimplification. Firms may be concerned with maximizing revenues, market shares, sales (Baumol, 1958) or even engage in satisficing behavior (Simon, 1957). The complexity of managerial decisions and the clear separation between ownership and management are frequently cited as the main obstacles to the achievement of true profit maximization.

Another overlooked aspect of competitive behavior is that the participants care not only about their absolute performance, but also consider their performance relative to that of their competitors. Modern society emphasizes the judgment of people on a relative scale. We label people as successful or talented based on their performance relative to their peer group rather than in relation to some absolute scale. From an early age people are exposed to the concept of grading on a curve. The student who gets an 'A' is the one that outscored the rest of the class not necessarily the one who could answer every question correctly. Conversely, one may do rather well objectively and still be viewed as a relative failure. There is also ample evidence that athletes prefer to win an important tournament or an Olympic event rather than hold the world record, again exhibiting clear preference for relative excellence.

Implicit in the profit maximization paradigm is the idea that managers care only about their own absolute performance, and consequently that they are unconcerned with how their performance compares to that of their rivals. It is exactly this assumption that we would like to challenge. Why should managers of a firm be any different than other people? We argue that they are not. Business people are likely to be naturally rivalrous, caring more about relative position and status than about absolute profits. This view of managers' motivation is supported by the popular business literature. For example, in his book *Competitive Advantage*, Porter argues that one of the two fundamental tasks in establishing and maintaining a successful firm is that of identifying 'the determinants of relative competitive position

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within an industry' (Porter, 1985, p. 1). Consequently, according to Porter, successful firms and successful managers are concerned with relative performance.

This paper examines the way in which standard models of competition change once it is recognized that managers may be concerned with some combination of relative and absolute performance. We assume that managers have a variety of different attitudes toward relative performance, and that each manager's attitude is captured by his[1] 'type'. In terms of their behavior, each manager will try to optimize some combination of his own profits and the rival firm's profits; his type determines the relative weights. Aggressive managers will put more weight on the difference between own profits and their rival's profits. Very cooperative managers may even put positive weight on the profits on their rivals.

We do not intend to challenge the view that the objective of the firm and its owners is to maximize profits. Rather, we would claim, as have many others, that it is possible for the management to have different objectives than the ownership. These differing objectives, as manifested in attitudes toward relative performance, may result from the personality of managers, their education and leadership style, or attention to books such as Porter's. The main point is that the manager is committed to his own type, and by hiring the manager the owner has the opportunity to commit her firm to relative performance. Thus, the attitude of managers is a tool that should be used by profit maximizing firms in order to influence their competitive posture. This insight would seem to hold true whether managers differ over their attitudes toward relative performance or any other inter-firm dimension.

To illustrate how the equilibria of market games change when the managers care about relative performance, consider the following simple example. Two firms compete in Cournot game with inverse demand $P = 1 - q_1 - q_2$. Each firm has zero marginal cost. The manager of firm 1 is a conventional profit maximizer, while the manager of firm 2 maximizes the difference between firm 2's profit and the profit of firm 1. It is easy to verify that in the unique equilibrium the quantities produced are $q_1^* = \frac{1}{4}$ and $q_2^* = \frac{1}{2}$. In other words, firm 2 gets the payoff of the Stackelberg leader and firm 1 gets the payoff of the Stackelberg follower in standard textbook Cournot competition. By hiring an 'aggressive' manager, firm 2 gets its best possible payoff.

However, if we instead supposed that firm 1 also hires a difference-maximizing manager, then in equilibrium both firms produce the competitive quantity of $\frac{1}{2}$, driving the price to zero and earning both firms zero profit. Hence the equilibria of these games can change drastically once we recognize that managers may possess different types.

The bulk of this paper derives the equilibria in various competitive environments for the two stage manager selection process in which owners first choose what type of manger to hire, and the managers then compete against each other. We start by showing that, given the possibility to hire managers who care about relative performance, firms will do so. In Cournot competition, the best manger against a firm that is purely profit maximizing is one that is purely relative difference maximizing. In quantity competition, if one firm is allowed to hire its manager first, that firm will behave more aggressively and make a larger profit compared to the simultaneous Cournot outcome.

Next we investigate the effects of making the manger's type endogenous to the selection process. When managers compete in quantities we show that the firm with the lower production costs will hire the more aggressive manager (the one that puts more weight on difference maximizing). The result is more competitive than standard Cournot behavior (Profits are lower, quantities produced, and welfare are higher). In a differentiated products competition model, we show that if the firms compete with strategic substitutes (price competition with complementary goods or quantity competition with substitute goods), the firms become more aggressive and total profits decrease. While if the firms compete with strategic complements (price competition with su goods or quantity competition with complementary goods), the firms become more cooperative in equilibrium, and total profits increase.

Immediately following this section is a literature review. Section 3 describes the general model. In Section 4 we investigate quantity competition. Section 5 considers quantity competition when one firm can hire its manager first. Price and quantity competition in a differentiated products duopoly are discussed in Section 6. Section 7 concludes. Most of the proofs are contained in an appendix following Section 7.

RELATED LITERATURE

While the vast majority of oligopoly models focus exclusively on profit maximizing firms, interpersonal comparisons of income, status, and utility have received considerable attention by economists, psychologists and sociologists. Duesenberry (1949) formulated his famous relative income hypothesis, stating that saving rates depend on a family's percentile position in the income distribution rather than on the actual money earned. Hirsch (1976) further emphasized the role of relative social status in economic decision making. Kapteyn and Wansbeek (1982) argue that personal utility is completely relative. They claim that people view their own well being only in relation to others.

A recent body of literature investigates the effects of delegation and distortion of managerial preferences on the competitive performance of firms. It explicitly considers profit maximizing firms which hire managers who do not profit maximize. Fershtman and Judd (1987) and Sklivas (1987) independently show that profit maximizing firms will sometimes choose to distort the preferences of their managers away from profit maximization. They consider two stage duopoly models where in the first stage profit maximizing firms choose compensation schemes for managers that are a linear combination of profits and sales. In the second stage, the managers, knowing both compensation schemes, compete in a duopoly. In particular, Sklivas shows that in Cournot competition, owners will choose to have managers put less than full weight on their costs, while in differentiated products price competition with substitute goods, firms will put more than unitary weight on their costs. This is equivalent to putting positive weight on both profits and sales in the first case, and putting positive weight on profits but negative weight on sales in the second. The results of Fershtman and Judd are similar in character.

Vickers (1985) is concerned with determining conditions under which a firm in a two stage duopoly game will choose to have its manager maximize profits. Vickers also considers a model where firms compensate managers based on a linear combination of profits and sales. However, he mentions in passing that a similar analysis should be done with a firm compensating managers based on a combination of own profits and the total profits of competing firms. This line of research is further pursued in this paper.

We investigate a two stage oligopoly competition environment in the style of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) (FJSV). In the first stage the owners of profit maximizing firms choose a manager whose personality (type) fits their competitive goals, where the type of the manager indexes his attitude toward relative performance. Specifically, a manager of type ϕ cares about his own profit minus ϕ times the profit of his rival. In the second stage the managers (who know each other's type) engage in an oligopoly competition in prices or quantities a la Cournot or Bertrand.

One key difference between our model and the FJSV model is that here each manager explicitly takes the other firm's performance into account when making his decision. As a result, our model captures the idea that managers may be concerned with relative performance too, and that the ability to commit to caring about relative performance may be of strategic value of the firm.

A second important difference between the approach taken here and the managerial incentive approach of FJSV is that we consider the problem faced by the firm to be that of choosing the best type of manager from among all of the types of managers that are available. Thus, commitment in our model stems from the choice of manager and the fact that managers are committed to behaving in a certain manner by virtue of their personality types. In contrast, commitment in the FSJV model comes from the manager's compensation scheme and his desire to maximize his own compensation. While the difference is somewhat semantic, and indeed, commitment in our model could be explained in terms of a contract that compensates managers on the basis of both absolute and relative performance, there is a certain insight that is captured only by our approach. Namely, the personality of the manager and his attitude toward competition may be of strategic value, and that the firm can publicly commit to this type of behavior simply by hiring the manager.

Our formulation also allows us to avoid a slight technical problem. In the managerial incentive approach, the net payoff to the firm is the profit the firm makes less the compensation paid to the manager. Since the manager's compensation is tied to performance, the incentives to the firm are slightly distorted from true profit maximization. If profits are large relative to the manager's compensation, this distortion is small. Nevertheless, our approach avoids this problem altogether by assuming that commitment comes through the manager's type and that compensation is independent of performance.

Further, our formulation permits us to consider firms that prefer their managers to maximize a weighted sum of own profits, Π_i, and the profits of its rivals, $U_i^{\phi} = \Pi_i + \phi \cdot \Pi_{-i}$. While anti-trust issues make it unlikely that a firm could compensate its manager with a scheme based on U_i^{ϕ} with $\phi > 0$ especially if all managers in the industry are similarly compensated. There is nothing to stop the firm from hiring a manager whose management philosophy accords with U_i^{ϕ}, i.e. he believes that 'a healthy industry is essential to a healthy firm'. The equilibria of some of the duopoly competition forms to be considered will lead to positive ϕ, and consequently it is important that our model provide a compelling explanation for them.

Both our paper and the FJSV type models assume that the managers' preferences become common knowledge before they compete against each other. This enables the contracts to function as commitment devices. Katz (1991) characterizes general conditions under which unobserved agency contracts can serve as commitment mechanisms.

Several papers have attempted to extend the results of Fershtman and Judd, Sklivas and Vickers. Fumas (1992) investigates relative performance compensation schemes as a control mechanism for the unobservable effort of risk averse managers. He shows that by looking at relative performance, risk neutral owners can reduce expected salaries paid to risk averse managers. Hwang and Mai (1995) consider a general conjectural variation version of the FJSV model in which they show that 'owners make their managers behave more (less) aggressively and produce more (less) than profit-maximizers if the managers' conjectural variations with respect to outputs are larger (smaller) than the actual response'. Basu (1995) shows that if the owner must explicitly choose whether to incur a cost to hire a FJSV-style manager or represent himself in the second stage game (as a profit maximizer), the Stackelberg equilibrium can occur as the result of this competition. Fershtman *et al*. (1991) consider a general model of strategic delegation and show that if the principals (owners) and agents can sign contracts that can be conditioned upon the compensation scheme of the opposing agent (manager), every Pareto optimal outcome of the game becomes a subgame perfect Nash equilibrium of the delegation game. In the context of Cournot-type games, this implies that the collusive outcome is an equilibrium of the two stage game.

Donaldson and Neary (1984) suggest that relative profit maximization might be used to alter the incentives of firms. In a 'socialist industry' where all the firms are government owned, efficient outcomes with minimum supervision can be achieved (in various environments) by setting firms goals to maximize $U_i = \Pi_i - 1/(n-1)\Sigma_{j \neq i} \Pi_j$ where n is the number of firms in the industry. Lundgren (1996) extends their idea to putting general weights on rival profits. He presents a comprehensive implementable program to prevent collusion in an oligopoly setting by the clever manipulation of the weight by a government or a central planner. He shows that if the weights are such that firms are in fact engaged in a zero sum game, their competition will be fierce and the total welfare to consumers optimal. Note however, that both of those papers deal with exogenous weights on relative performance.

Finally, Gibbons and Murphy (1990) show empirically that managers' compensations are based not only on absolute measures but on relative performance as well. There is also ample anecdotal evidence that top managers' compensation schemes are tied to the overall performance of the company in the industry and its stock price. Thus, managers have an incentive if not a necessity to constantly compare their achievement to the industry's benchmark.

THE BASIC MODEL

We consider a two-stage owner–manager duopoly competition. Following Fershtman and Judd (1987), when we say 'owner' we mean an individual or a group whose sole purpose is to maximize the profits of the firm. 'Manager' refers to an agent that the owner hires to make real time operating decisions. Potential managers take on a continuum of attitudes toward relative performance which is captured by their type, ϕ. The objective function (utility function) of a manager of type ϕ puts weight of $(1-\phi)$ on own profits and a weight ϕ on the difference between own profits and the profits of the firm's rival. This is equivalent to putting unit weight on own profits and weight $-\phi$ on the rival's profit. Hence we can write the objective function of a type ϕ manager working for firm i as:

$$U_i^{\phi} = \Pi_i - \phi \cdot \Pi_{-i}.$$

When the objective function is written in this manner, it becomes apparent that if $|\phi| > 1$, the manager is more concerned with his rival than with the performance of his own firm. Since it is unreasonable to believe that managers actually behave in this manner, we limit the range of possible 'types' to the interval $\phi \in [-1, 1]$. However, the model is rich enough to allow for the possibility that $|\phi| > 1$.

Throughout the paper, we will refer to our two-stage model as the Manager Selection Process (MSP). In the first stage of the MSP, each owner hires a manager that fits her competitive goal of maximizing profits. We assume that managerial compensation is independent of type. One can view the market in the following way: managers walk around offering their services to various firm owners who then choose the one most suitable for them. A different interpretation of the first stage (similar to Fershtman and Judd's (1987)) would be that firms can make a take-it-or-leave-it offers to managers tying the compensation to a combination of own and rival's profits. Thus the compensation scheme effectively determines the behavior of the manager. If we assume the effect on owner's profit of tying managerial compensation to firm performance is negligible, then these approaches are essentially equivalent.

At the beginning of the second stage the types of the managers are revealed and become common knowledge (perhaps through contact at unmodeled cocktail parties or through previous interactions). Then, the two managers engage in some form of duopoly competition.

Schematically, the game proceeds as follows:

Stage 1. The owner of firm j hires a manager of type ϕ_j, with the goal of maximizing profit given the managerial choices of the other firms and knowing the type of competition they will face (i.e. price, quantity, etc.)

Stage 2a. The types of the managers become common knowledge to all of the other managers.

Stage 2b. The managers compete in some sort of duopoly competition.

In Section 4, the firms compete in (simultaneous) Cournot competition with quadratic costs. In Section 5 we consider Stackelberg (sequential hiring of managers) competition in the product market. Section 6 considers several types of differentiated products duopoly with constant costs. Section 6.1 considers price competition while Section 6.2 compares price and quantity competition under the same demand system.

Once the types of the managers have been determined, there is a unique equilibrium in stage 2 of the game for all of the forms of competition considered here. Because of this, it is reasonable to expect owners to make their decision based on the premise that the managers will play the unique equilibrium in the second stage. Hence the equilibrium concept we employ is that of subgame perfection. A subgame perfect equilibrium (SPE) in this context requires that for any fixed pair of managerial types, each manager plays a best response to his opponent's strategy in the second stage. Furthermore, each owner in the first stage must choose a type of manager that is a best response to the manager selection of the other owners.

As an illustration, consider Cournot competition with linear demand ($P = 1 - q_1 - q_2$) and no production costs. For fixed managerial types (ϕ_1, ϕ_2) an equilibrium of the second stage quantity competition is given by (for $i = 1, 2$):

$$q_i(\phi_1, \phi_2) = \frac{1 + \phi_i}{3 + \phi_1 + \phi_2 - \phi_1 \phi_2}.$$

Substituting into the firm's profit function and solving for the optimal managerial types yield the following equilibrium:

$$\forall \phi_1^* \in [0, 1], \quad \phi_2^* = \frac{1 - \phi_1^*}{1 + 3\phi_1^*},$$

$$q_1 = \frac{1}{4} \cdot \frac{1 + 3\phi_1^*}{1 + \phi_1^*}, \quad q_2 = \frac{1}{2} \cdot \frac{1}{1 + \phi_1^*}.$$

Note that while there exists a unique equilibrium in the second stage, we actually found a continuum of equilibria for the entire game. All of those have total quantity produced at $\frac{3}{4}$, market price at $\frac{1}{4}$, and the total industry profits at $\frac{3}{16}$ (profits are given by: $\Pi_1 = \frac{11}{16}((1 + 3\phi_1^*)/(1 + \phi_1^*)$, $\Pi_2 = \frac{1}{8}1/(1 + \phi_1^*))$.

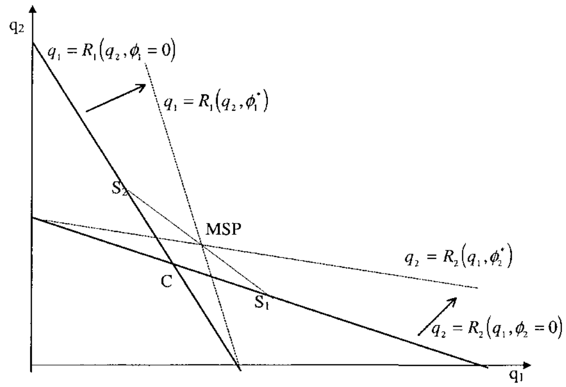

Figure 1. The solid lines represent the reaction curves for the standard Cournot competition. The dashed lines represent the MSP reaction curves, these are the previous reaction curves rotated through the intercept. Finally, the set of all MSP equilibria is represented by the dotted line connecting the equilibrium where firm 1 is the Stackelberg leader (S_1) to the one where firm 2 is the leader (S_2).

The equilibria differ only in the way these quantities and profits are divided among the firms. Each firm may produce as much as the Stackelberg leader quantity ($\frac{1}{2}$) and as little as the Stackelberg follower quantity ($\frac{1}{4}$). Moreover, the weights used in the example from the introduction correspond to those extreme equilibria.

The unique symmetric equilibrium of this game has $\phi_1 = \phi_2 = \frac{1}{3}$. In this equilibrium each firm produces $q = \frac{3}{8}$ and earns profit $\Pi = \frac{3}{32}$. Hence the firms do slightly worse than in the standard Cournot model, where they each earn $\frac{1}{9}$.

A graphic illustration (Figure 1) might shed some light.

COURNOT COMPETITION

In this section we consider an application of the managers selection process to a case of Cournot competition with convex costs in a duopoly setting. Specifically, after the simultaneous selection of their managers, both firms conjecture that the other's quantities are fixed regardless of their own actions. The following results hold for a general downward sloping demand monotone inverse demand functions. For the sake of expositional clarity and since no new insight is gained by considering the more general demand system, we will use the simpler inverse demand function, $P = 1 - q_1 - q_2$. We also take the simplest convex cost function such that the cost for firm i is given by $C_i(q_i) = c_i q_i^2$.

The method for finding the MSP equilibrium with any competitive environment is backward induction. We present it in detail here.

Begin with the second stage. The managers known types (ϕ_1, ϕ_2) determine the equilibrium quantities (for $i = 1, 2$):

$$q_i(\phi_1, \phi_2) = \frac{1 + 2c_j + \phi_i}{3 + 4(c_1 + c_2 + c_1 c_2) + \phi_1 + \phi_2 - \phi_1 \phi_2}.$$

Notice that $q_i(\phi_1, \phi_2)$ is an increasing function of ϕ_i and a decreasing function of ϕ_j thus by hiring a more aggressive manager the owner can increase the quantity that is produced by her firm and decrease the quantity produced by the rival.

We now turn our attention to the first stage, where profit maximizing owners choose what type of managers to hire, given the equilibrium play in the second stage. We omit the straightforward calculations and summarize the results in the following proposition.

Proposition 1:
If the inverse demand function is given by $P = 1 - q_1 - q_2$, and the production cost for firm i is given by $C_i(q_i) = c_i q_i^2$. The unique equilibrium to the manager selection process is characterized by:

$$\phi_i = \frac{c_j}{2c_i + 2c_i c_j + c_j},$$

$$q_i = \frac{1}{4} \frac{2c_j + 2c_i c_j + c_i}{(1 + c_i)(c_i + c_j + c_i c_j)},$$

$$\Pi_i = \frac{1 + 2c_j}{16} \cdot \frac{2c_j + 2c_i c_j + c_i}{(1 + c_i)(1 + c_j)(c_i + c_j + c_i c_j)}.$$

This characterization of the equilibrium allows us to make the following observations.

Corollary 1:
The firm with the lower production costs will hire the more aggressive manager (the one that puts more weight on difference maximizing[2]

Corollary 1 implies that if the types of the managers are known, this information can be used to make inferences about the relative costs of the firms.

It is natural to compare the equilibrium in the MSP to the equilibrium in a standard Cournot competition, where both managers are profit maximizers. This is the subject of the next proposition.

Proposition 2:
When the managers' selection process is compared with Cournot competition:
1. Total profits are smaller.
2. Total quantity produced is larger.
3. Total welfare is higher.

Hence as a result of the manager selection process, the equilibrium more closely resembles the competitive equilibrium than the standard Cournot equilibrium. It is important to note that the fact that profits are smaller in the MSP is an equilibrium phenomenon. Each firm is responding optimally to the other firm's choice of manager, and any unilateral deviation to a different managerial choice such as pure profit maximization would result in the firm earning even lower profits. Furthermore, since both firms selecting pure profit maximizing managers is not an equilibrium, the optimal response to a pure profit maximizing rival is to hire a manager who does not maximize profit. In fact, in this environment, the optimal response to a pure profit maximizing rival is to hire a pure difference maximizing ($\phi = 1$) manager.

Since the MSP results in lower aggregate profits, it may at first glance appear undesirable for the companies. However, this is not always the case. The next two propositions describe conditions under which one of the firms may benefit. We assume without loss of generality that firm 1 has higher costs of production ($c_1 > c_2$).

Proposition 3:
Under the MSP, the quantity produced by firm 2 and its market share are always larger than in a standard Cournot case. Furthermore, if firm 2 has a big enough cost advantage $\left(c_2 < c_1/(2c_1+2)\right)$ then the quantity produced by firm 1 is lower.

This shows that the MSP is socially desirable in the sense that the firm with the lower cost actually produces more and has a larger market share than a Cournot competition. The conditions under which the low cost firm actually stands to benefit from using MSP are stated in Proposition 4.

Proposition 4:
If firm 2 has a low cost $\left(c_2 < \frac{1}{4}\right)$ and a big enough cost advantage $\left(c_1 > (c_2(8c_2+7))/2(c_2+1)(1-4c_2)\right)$ then it generates more profit in the MSP than in the Cournot competition case. Firm 1's profit is always lower.

Hence firms with low costs will tend to hire aggressive managers in order to exploit their relative advantage to the maximum. These firms tend to gain larger profits in the MSP case than by competing in a standard Cournot competition. This is in contrast to Sklivas, for example, where firms are always better off engaging in a standard Cournot competition.

The next proposition generalizes the previous results to n-firm competition with equal costs. Firm i produces q_i units at a cost of $C_i(q_i) = cq_i^2$. Market price is given by $P = 1 - \sum_{i=1}^{n} q_i$ and the objective function of the manager of firm i is given by:

$$U_i^{\phi} = \Pi_i - \phi_i \cdot \sum_{j\neq i} \Pi_j = Pq_i - C_i(q_i) - \phi_i \times \sum_{j\neq i} \left(Pq_j - C_j(q_j)\right)$$

$$U_i^{\phi} = P(q_i - \phi_i \cdot q_{-i}) - c\left(q_i^2 - \phi_i \sum_{j\neq i} q_j^2\right),$$

where $q_{-i}\ \sum_{j\neq i} q_j$

Proposition 5:
There exists a unique symmetric equilibrium in the above n-firm MSP with equal costs. This equilibrium is characterized by:

$$\phi_{Cournot}^{MSP} = \frac{2c+n+1-w}{2n-4},$$

$$q_{Cournot}^{MSP} = \frac{2n-4}{n^2+2cn-2n+nw-3-w-6c},$$

$$\Pi_{Cournot}^{MSP} = (4-2n)\frac{n^2-2n-nw+3+2c+w}{(n^2+2cn-2n+nw-3-w-6c)^2},$$

where $w = \sqrt{4c^2+4cn+4c+n^2-2n+9}$

Corollary 2:
As the number of firms increases the optimal ϕ_i decreases and approaches zero. Hence, as the number of firms in the market increases, the type

of manager chosen in equilibrium approaches pure profit maximization.

Note that for an n-firm symmetric Cournot competition the quantities produced and profits per firm are:

$$q_{Cournot} = \frac{1}{2c+n+1} \quad \Pi_{Cournot} = \frac{c+1}{(2c+n+1)^2}.$$

It is easy to see that

$$\lim_{n\to\infty} \frac{q^{MSP}_{Cournot}}{q_{Cournot}} = \lim_{n\to\infty} \frac{\Pi^{MSP}_{Cournot}}{\Pi_{Cournot}} = 1.$$

Thus, as n increases the quantities and profits in the MSP approach those of Cournot, as is to be expected.

SEQUENTIAL MANAGER SELECTION PROCESS

In this section we consider a duopoly where one firm is able to move first and hire a manager. In the first stage, firm 1 chooses its manager. In the second stage, firm 2 observes the managerial choice made by firm one and then chooses its manager. In the third stage managers engage in Cournot competition with quadratic costs. As before, the equilibrium is derived via backward induction. The tedious but straightforward calculations can be found in the appendix, the equilibrium is characterized in the next proposition.

Proposition 6:
In a two firm competition where firm 1 hires first. If its costs are big enough ($c_1 > 0.5$) there exist a unique equilibrium to the manager commitment environment with the following managerial selection

$$\phi_1 = \frac{2c_1 + 4c_1c_2 + 4c_2 + 1}{4c_1^2(c_2+1) + 6c_1c_2 + 4c_1 + 2c_2 - 1},$$

$$\phi_2 = \frac{2c_1 - 1}{2c_1 + 4c_1c_2 + 4c_2 + 1}.$$

Furthermore, for fixed production cost c_1 for firm 1, as the production costs of firm 2 increase it will search for a more and more neutral $\phi_1 \to 0$ manager. At the same time the optimal manager obtained by firm 1 becomes more and more aggressive $\phi_1 \to 2/(2c_t + 1)$. Hence as the gap between the costs of firms grows, firm 1 becomes more able to take advantage of its leadership position.

Note that if the cost of production for firm 1 is low enough ($c_1 < 0.5$) the optimal managerial choice will involve putting more (negative) weight on the opponent's profit than the firm's own, setting $\phi_1 > 1$. Since we have limited the range of possible types, firm 1 will hire a true difference maximizing manager $\phi_1 = 1$. Firm 2 (regardless of its own production cost) will be forced to hire a neutral manager $\phi_1 = 0$, and behave as a true profit maximizing firm. In this case, firm 1 gets the Stackelberg leader profits, and firm 2 settles for the follower outcome. Thus, low costs imply that hiring first is equivalent to producing first in a standard Stackelberg setting.

The next proposition compares the sequential manager selection process (SMSP) to the previous managers' selection process (MSP). The results are intuitive. The firm that hires its manager first will seek a more aggressive manager, produce more and will make larger profits than in the simultaneous hiring case. The second firm will correspondingly seeks a less aggressive manager, produce less and will make less money.

Proposition 7:
Under the SMSP regime the firm that chooses its manager first produces larger quantities and makes higher profit than under the MSP. It also chooses a more aggressive manager. The converse is true for the firm selecting last. Furthermore, SMSP leads to higher total industry production and lower total profits.

An illustrative example (Table 1) might help clarify the relationship between Cournot, MSP and SMSP: Consider the case where $c_1 = 1$ and $c_2 = 0.1$.

The collusive outcome is provided to illustrate the optimal allocation of output among firms. The costs in this case satisfy the conditions of propositions 3 and 4. As expected, firm 2's quantity and profit are larger in the MSP than in the standard Cournot case. Note that in the MSP, both firms overproduce relative to the collusive allocation, while in the Cournot, only the high cost firm overproduces. SMSP(1) refers to the situation where firm 1 hires its manager first, since $c_1 > 0.5$ the results are characterized by Proposition 6. Notice that Firm 1's quantity and profit are

Table 1. Numerical Example

	Collusive	Cournot	MSP	SMSP(1)	SMSP(2)
q_1	0.04167	0.15385	0.14583	0.19792	0.13636
q_2	0.41667	0.38462	0.43561	0.38826	0.45455
q_1+q_2	0.45834	0.53847	0.58144	0.58618	0.59091
Π_1	0.02083	0.04734	0.03977	0.04273	0.03719
Π_2	0.20833	0.16272	0.16335	0.14560	0.16529
$\Pi_1+\Pi_2$	0.22916	0.21006	0.20312	0.18833	0.20248
ϕ_1			0.04348	0.46341	0
ϕ_2			0.71429	0.26316	1

increased relative to the MSP. Conversely, Firm 2's profit and quantity decrease. The SMSP(2) column describes the results when firm 2 hires first. Since $c_2 = 0.1 < 0.5$ the firm will hire a difference maximizing manager $\phi_2 = 1$, and get the Stackelberg leader outcome. When compared to MSP, quantity and profit are larger for firm 2 and smaller for firm 1.

Finally, we compare the results of the SMSP to those of the MSP when firms with symmetric costs are considered.

Proposition 8:
If both firms have the same production costs: $C_i(q) = cq^2$ then:

$$q_{1-SMSP} > q_{MSP} > q_{2-SMSP},$$
$$\Pi_{1-SMSP} > \Pi_{MSP} > \Pi_{2-SMSP},$$
$$\phi_{1-SMSP} > \phi_{MSP} > \phi_{2-SMSP}.$$

It is worthwhile to note that in the above case the firm that moves first will always hire a more aggressive manager.

DIFFERENTIATED PRODUCTS

The next environment we consider is differentiated products duopoly with linear demand and constant marginal cost. As noted in Spulber (1989) and Singh and Vives (1984), in such environments the outcome depends on whether the firms compete in prices or quantities, and on whether the goods are substitutes or complements. Consequently we consider two different models here. Section 6.1 is concerned with price competition. Section 6.2 deals with the corresponding quantity competition and compares the equilibrium values for the same demand system.

Price Competition

In this section we consider a model of differentiated products duopoly where the firms compete in prices. Demand for the output of firm i is given by the expression $q_i = 1 - p_i + zp_j$ where $|z| \leq 1$ and the firms have no production costs.[3] Once again, we use backward induction and consider a two-stage process where firms choose managers in the first stage and compete in the second stage. We continue to refer to this as the Manager Selection Process.

Proposition 9:
Under the specified differentiated products price competition, the MSP possesses a unique equilibrium that is characterized by the following:

$$\phi^P_{MSP} = \frac{z}{z-2}$$
$$P^P_{MSP} = \frac{1}{4}\frac{z-2}{z-1} \qquad q^P_{MSP} = \frac{1}{4}(2+z)$$

Corollary 3:
The sign of ϕ^P_{MSP} is the opposite of that of z. In other words when the goods are complements ($z < 0$), the firms become more aggressive, setting $\phi^P_{MSP} > 0$. Conversely, when the goods are substitutes the firms become more cooperative $\phi^P_{MSP} > 0$.

Thus, in situations when the goods are complements, the MSP makes the competition less aggressive. Conversely, when the goods are substitutes the MSP increases competition.

Note that in the standard differentiated price competition equilibrium $p_i = q_i = 1/(2-z)$ and $\Pi_i = (1/(2-z))^2$.

Proposition 10:
When compared with the standard price competition model (with $\phi_1 = \phi_2 = 0$), profits are larger (smaller) in the manager selection game than in the standard game whenever the substitutability parameter z is positive (negative). Furthermore, the quantity produced by each firm is always smaller and the price charged is always higher.

Comparison of Price and Quantity Competition

In this subsection we consider differentiated products quantity competition with linear demand. It seems natural to investigate a system of the form $p_i = 1 - q_i + wq_j$ where $|w| \leq 1$ without production costs. But this formulation is completely analogous to the one considered in Section 6.1 as a matter of fact the equilibrium values of price, quantity, and profits for the standard case are *identical* in the two cases.

In order to truly compare how the Manager Selection Process affects price and quantity competition, we must derive equilibria for the *same* demand system under both conjectures. We begin by deriving the equilibrium for the quantity competition for the exact demand system used in Section 6.1 where the inverse demand functions are given by $p_i = 1/(1-z^2)((1+z) - q_i - zq_j)$. Using the same method as before we determine the equilibrium values.

Proposition 11:
The unique equilibrium of the Manager Selection Process is characterized by the following:

$$\phi^q_{MSP} = \frac{z}{z+2},$$

$$P^q_{MSP} = \frac{1}{4}\frac{z-2}{z-1}, \qquad q^q_{MSP} = \tfrac{1}{4}(2+z).$$

Corollary 4:
The sign of ϕ^q_{MSP} is same as the sign of z. Hence, in contrast to the quantity competition case, when the goods are complements ($z > 0$) and the firms compete on quantity, they are more aggressive in the MSP. Conversely, when the goods are substitutes and the firms compete on quantity, they are less aggressive in the manager selection process.

Proposition 12:
When compared to the equilibrium of the standard differentiated products duopoly quantity competition model (with $\phi_1 = \phi_2 = 0$), profits are larger (smaller) in the manager selection process whenever z is less (greater) than zero. Furthermore, in the MSP quantities are larger and prices are smaller than in the standard case.

The following propositions compare the equilibria in price competition (Proposition 9) and quantity competition (Proposition 11), which involve the same demand system under price and quantity competition, respectively. Recall that equilibrium values under price competition have superscript p, while equilibrium values under quantity competition have superscript q.

Proposition 13:
The equilibrium values of ϕ^p_{MSP} and ϕ^q_{MSP} are of opposite signs. In particular, ϕ^p_{MSP} is positive (negative) and ϕ^q_{MSP} is negative (positive) whenever z is less (greater) than zero.

In price competition with substitute goods, prices are strategic complements, while in price competition with complementary goods, prices are strategic substitutes. Conversely, in quantity competition with substitute goods, quantities are strategic substitutes, and in quantity competition with complementary goods, quantities are strategic complements. The following corollary summarizes.

Corollary 5:
If the firms compete with strategic substitutes, the optima1 managerial choice has $\phi > 0$ and total profits decrease due to the MSP. If the firms compete with strategic complements, the optimal managerial choice has $\phi < 0$ and the MSP increases total profits.

Thus, when firms compete in strategic substitutes, MSP lessens competition and the outcome becomes more cooperative. On the other hand, when firms compete in strategic complements, MSP increases the competitiveness of the firms. The results are summarized in Table 2. The

Table 2. Comparison between Different Types of Competition

Competition	Goods	Equivalent to strategic	Sign of ϕ_{MSP}	Δp_i	Δq_i	$\Delta\Pi_i$
Price	Substitutes	Complements	–	+	–	+
Price	Complements	Substitutes	+	+	–	–
Quantity	Substitutes	Substitutes	+	–	+	–
Quantity	Complements	Complements	–	–	+	+

changes in the last three columns are computed by subtracting the *MSP* value from the standard value.

Recall that in the standard model with substitute goods, price competition is fiercer than quantity competition. Since under the MSP, quantity competition becomes more intense, while price competition becomes less fierce when the goods are substitutes, we expect the differences between the two models to decrease as indeed it does. Furthermore, Miller and Pazgal (2001) show that if owners have sufficient control over their managers' incentives, such as relative performance compensation provides, then the equilibrium outcome (prices, quantities, and profits) of a two-stage delegation game will not depend on whether the managers ultimately compete in prices or in quantities.

CONCLUSION

This paper explicitly addresses the fact that managers' attitudes toward relative performance can affect their competitive behavior. Consequently, such attitudes could be of strategic value to profit maximizing firms. In order to illustrate this point, we derive the equilibria of various duopoly and oligopoly environments where the manager type is endogenous.

We would like to emphasize two important points. The first is that it can be useful for profit maximizing firms to hire managers that care about relative performance. This can be accomplished through hiring managers whose personalities or managerial philosophies make them pay attention to relative performance, or by tying managerial compensation to relative performance (as it becoming increasingly common for public firms). As the models studied above show, recognition of this fact can in some cases lead to increased profit performance for the firm. The most dramatic example is the case of Cournot competition with constant marginal cost, where we show that if a firm hires a pure difference maximizing ($\phi = 1$) manager in response to a profit maximizing rival manager, it can achieve the Stackelberg leader payoff.

The second point is that the personality or type of a manager can be used as a strategic commitment device. While the FJSV models assume that the preferences of the manager are dictated by his compensation scheme, we assume that this preferences are predetermined by their type. By hiring the manager, the firm credibly commits to a certain attitude toward relative performance.

There is significant evidence that managers do, in fact, care about relative performance. This evidence comes from areas such as psychology, empirical economics, and the popular management literature. However, if managers do not differ in their attitude toward relative performance but on some other personality dimension, the arguments in this paper suggest that the firm should take this into account. If the dimension is relevant to the preferences of the manager in the competitive environment, then this is of strategic importance to the firm. If the dimension is not important, this knowledge is of value to the firm, because it implies that rival managers' types should not affect their competitive posture. Thus we argue that the personality of the manager can and should be taken into account by the firms when making hiring decisions.

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APPENDIX A

Proof of Corollary 1:
In equilibrium the difference between the managers' aggressiveness is:

$$\phi_1 - \phi_2 = 2(c_1c_2 + c_1 + c_2)\frac{c_2 - c_1}{(2c_1 + 2c_1c_2 + c_2)(2c_2 + 2c_1c_2 + c_1)}\phi$$

which is positive if $c_1 < c_2$. □

Proof of Proposition 2:
The Cournot equilibrium is characterized by:

$$q_i = \frac{1 + 2c_j}{3 + 4c_i + 4c_j + 4c_ic_j},$$

$$\Pi_i = \frac{(1 + 2c_j)^2(1 + c_i)}{(4c_j + 4c_ic_j + 3 + 4c_i)^2}.$$

All the above observations are proved by direct comparison.

The total profits in Cournot competition are:

$$\frac{1}{16}(c_2 + c_1)\frac{4c_1c_2 + 4c_1 + 4c_2 + 3}{(c_2 + 1)(c_1 + 1)(c_1c_2 + c_1 + c_2)}.$$

While the total profits in the MSP are:

$$\frac{4c_2^2 + 4c_1c_2^2 + 4c_1^2 + 4c_1^2c_2 + 8c_1c_2 + 2 + 5c_1 + 5c_2}{(4c_1c_2 + 4c_1 + 4c_2 + 3)^2}.$$

The difference is:

$$\frac{1}{16}\frac{36c_1c_2^2 + 5c_1 + 5c_2 + 4c_2^2 + 40c_1c_2 + 36c_1^2c_2 + 4c_1^2 + 32c_1^2c_2^2}{(c_2 + 1)(c_1 + 1)(c_1c_2 + c_1 + c_2)(4c_1c_2 + 4c_1 + 4c_2 + 3)^2},$$

which is always positive. The total quantity produced in a Cournot competition is:

$$2\frac{1 + c_2 + c_1}{4c_1c_2 + 4c_1 + 4c_2 + 3}.$$

The total quantity produced in the MSP is:

$$\frac{1}{4}\frac{2c_2^2 + 2c_1c_2^2 + 2c_1^2c_2 + 6c_1c_2 + 3c_2 + 2c_1^2 + 3c_1}{(c_1c_2 + c_1 + c_2)(c_1 + 1)(c_2 + 1)}.$$

The difference is:

$$\frac{1}{4}\frac{2c_2^2 + 2c_1c_2^2 + c_1 + c_2 + 2c_1c_2 + 2c_1^2c_2 + 2c_1^2}{(4c_1c_2 + 4c_1 + 4c_2 + 3)(c_1c_2 + c_1 + c_2)(c_1 + 1)(c_2 + 1)}$$

which is always negative.

Welfare is higher not only due to the fact that the total quantity produced is higher but also because the firm with the lower cost has increased its market share (as will be shown in the next proposition). □

Proof of Proposition 3:
Consider the difference in quantities for the firm with the lower cost (firm 2):

$$\frac{1}{4}\frac{2c_1 + 2c_1c_2 - c_2}{(1 + c_2)(c_1c_2 + c_1 + c_2)(3 + 4c_2 + 4c_1 + 4c_1c_2)}.$$

Since $c_2 < c_1 : 2c_1 + 2c_1c_2 - c_2 > 2c_1 + 2c_1c_2 - c_1 = c_1 + 2c_1c_2 > 0$ and the above expression is always positive.

The ratio firm 2's quantity produced to firm 1's quantity is: under MSP:

$$(c_1+1)\frac{2c_1+2c_1c_2+c_2}{(c_2+1)(2c_2+2c_1c_2+c_1)}$$

in a Cournot competition: $(1+2c_1)/(2c_2+1)$

The difference:

$$\frac{c_1-c_2}{(c_2+1)(2c_2+2c_1c_2+c_1)(2c_2+1)} > 0.$$

Thus firm 2's market share has increased.

The quantity produced by firm 1 in the MSP:

$$\frac{1}{4}\frac{2c_2+2c_1c_2+c_1}{c_1^2c_2+c_1^2+2c_1c_2+c_1+c_2}.$$

The quantity produced by firm1 in Cournot competition:

$$\frac{1+2c_2}{3+4c_2+4c_1+4c_1c_2}.$$

The difference:

$$\frac{1}{4}\frac{2c_1c_2-c_1+2c_2}{(c_1+1)(c_1c_2+c_1+c_2)(4c_1c_2+4c_1+4c_2+3)}$$

is negative as long as $2c_1c_2 - c_1 + 2c_2 < 0$ or as long as $c_2 < c_1/2c_1 + 2$. □

Proof of Proposition 4:

Profit for firm 1 in the MSP:

$$\frac{1}{16}(2c_2+1)\frac{2c_2+2c_1c_2+c_1}{(c_2+1)(c_1+1)(c_1c_2+c_1+c_2)}.$$

Profit for firm 1 in the Cournot case:

$$(2c_2+1)^2\frac{c_1+1}{(4c_2+4c_1c_2+4c_1+3)^2}.$$

The difference:

$$-\frac{1}{16}(2c_2+1)\frac{8c_1^2+6c_1c_2+7c_1+8c_1^2c_2-2c_2}{(c_2+1)(c_1+1)(c_1c_2+c_1+c_2)(4c_2+4c_1c_2+4c_1+3)^2}$$

is always negative.

(Since $c_2 < c_1 : 8c_1^2 + 6c_1c_2 + 7c_1 + 8c_1^2c_2 - 2c_2 > 8c_1^2 + 6c_1c_2 + 7c_1 + 8c_1^2c_2 - 2c_1 > 8c_1^2 + 6c_1c_2 + 5c_1 > 0$.)

Consider the difference in profits for firm 2, the one with the lower cost:

$$-\frac{1}{16}(1+2c_1)\quad\frac{8c_1c_2^2+6c_1c_2-2c_1+8c_2^2+7c_2}{(c_2+1)(c_1+1)(c_1c_2+c_1+c_2)(3+4c_2+4c_1+4c_1c_2)^2}$$

the expression is positive as long as

$8c_1c_2^2 + 6c_1c_2 - 2c_1 + 8c_2^2 + 7c_2 < 0$ or if $2(c_2+1)(4c_2-1)c_1 + 8c_2^2 + 7c_2 < 0$

which yields the desired result $c_1 > c_2(7+8c_2)/2(c_2+1)(1-4c_2)$. □

Proof of Proposition 5:
In the second stage (calculating quantities when the ϕ's are known) the first order necessary conditions for an equilibrium are:

$1 - 2(1+c)q_i + (\phi_i - 1)q_{-i} = 1$ for $i = 1, \ldots, n$,
where $q_{-i} = \sum_{j \neq i} q_j$.

For firm i, assume that all other firms use the same type manager ($\phi_j = \phi$ for $j \neq i$) and sum over all their first order conditions to get the following two equations:

$$1 - 2(1+c)q_i + (\phi_i - 1)q_{-i} = 0,$$
$$(n-1) - 2(1+c)q_{-i} + (\phi - 1)\left((n-1)q_i + (n-2)q_{-i}\right) = 0.$$

The solution yields q_i and q_{-i} as a function of the managers types.

Now firm's i problem is to find the best manager type, ϕ_i, to maximize its profit:

$$\Pi_i = \frac{-(\phi_i cn - \phi_i c + \phi n\phi_i - \phi\phi_i - 3c - 1 - 2c^2 - 2\phi - 2c\phi + \phi n + c\phi n)(2c + 1 - \phi_i + n\phi_i + 2\phi - \phi n)}{(-4c - n + \phi n - 3\phi - 4c^2 - 2cn + 2c\phi n - 4c\phi - n\phi_i - 1 + \phi_i + \phi n\phi_i - \phi\phi_i)^2}.$$

Differentiating Π_i with respect to ϕ_i, setting the derivative equal to zero and forcing all the managers to be of the same type yields the following result:

$\phi_i = \rho$ where ρ is a root of $(n-2)Z^2 + (-2c - n - 1)Z + 1$

Since we know that the sum of the roots is positive and their product is positive and less than one, we are guaranteed that the smaller root is positive and less than one.

If $n = 2$ the equation reduces to: $(-2c - 3)Z + 1 = 0$ or $\phi_i = 1/(3+2c)$ which is exactly the result we got for the two firm equal cost competition.

Explicitly, for $n \geq 3$ we have

$$\phi_i = \frac{1}{2}\frac{2c + n + 1 - \sqrt{(4c^2 + 4cn + 4c + n^2 - 2n + 9)}}{n-2}.$$

Substituting back we get the desired expressions for q_i, Π_i. □

Proof of Proposition 6:
We begin with the second stage. A manager for firm 1 of type ϕ_1 solves the following problem:

$$\max_{qi} (1 - q_1 - q_2)(q_1 - \phi_1 q_2) - c_1 q_1^2 - \phi_2 c_2 q_2^2.$$

The solution yields the following reaction function for manager 1:

$$q_1 = \frac{1 - (1 - \phi_1)q_2}{2 + 2c_1}.$$

Similarly, a manager of type ϕ_2 for firm 2 has reaction function:

$$q_2 = \frac{1 - (1 - \phi_2)q_1}{2 + 2c_2}.$$

Combining the two allows us to compute the following equilibrium for the second stage game:

$$q_2(\phi_1, \phi_2) = \frac{\phi_2 + 2c_1 + 1}{3 + 4c_2 + 4c_1 + 4c_1c_2 + \phi_1 - \phi_2\phi_1 + \phi_2},$$

$$q_1(\phi_1, \phi_2) = \frac{1 + 2c_2 + \phi_1}{3 + 4c_2 + 4c_1 + 4c_1c_2 + \phi_1 - \phi_2\phi_1 + \phi_2}.$$

The owner of firm 2 needs to choose her manager:

$$\max_{\phi_2 \in [-1,1]} (1 - q_1(\phi_1, \phi_2) - q_2(\phi_1, \phi_2))q_2(\phi_1, \phi_2) - c_2(q_2(\phi_1, \phi_2))^2$$

yielding reaction function:

$$\phi_2 = -\frac{-1+\phi_1-2c_1+2\phi_1 c_1}{4c_1c_2+4c_2+1+2c_1+3\phi_1+2\phi_1 c_1}.$$

Substituting into owner 1's problem we get

$$\max_{\phi_1\in[-1,1]} \frac{1}{16}(4c_2+4c_1c_2-2\phi_1 c_1+2c_1+\phi_1+1)$$

$$\times\frac{4c_1c_2+4c_2+1+2c_1+3\phi_1+2\phi_1 c_1}{2c_1^2c_2+2c_2+4c_1c_2+\phi_1+3c_1+2c_1^2+1+\phi_1 c_1)(2c_1c_2+2c_2+2c_1+\phi_1+1}.$$

With a maximum at:

$$\phi_1 = \frac{4c_1c_2+4c_2+1+2c_1}{4c_1^2c_2+4c_1^2+6c_1c_2+4c_1+2c_2-1}$$

substituting into the expression for ϕ_2 finishes the proof. □

Proof of Corollary 2:
We know that ϕ_i is positive.

$$\phi_i = \frac{1}{2}\frac{2c+n+1-\sqrt{(4c^2+4cn+4c+8+(n-1)^2)}}{n-2}$$

$$<\frac{1}{2}\frac{2c+n+1-\sqrt{((n-1)^2)}}{n-2}=\frac{1}{2}\frac{2c+2}{n-2}=\frac{c+1}{n-2}$$

Hence, for every $n\geq 2$ we have: $0\leq\phi_i<(c+1)/(n-2)$. □

Proof of Proposition 7: The proof uses simple comparisons:

1. $$q_{1\_SMSP}-q_{1\_MSP} = \frac{1}{8}\frac{2c_1+4c_1c_2+1+4c_2}{(c_1+1)(c_1+c_1c_2+c_2)} - \frac{1}{4}\frac{2c_2+2c_1c_2+c_1}{(1+c_1)(c_2+c_2c_1+c_1)}$$
$$= \frac{1}{8(c_1+1)(c_1+c_1c_2+c_2)}>0,$$

$$q_{2\_SMSP}-q_{2\_MSP} = \frac{1}{8}\frac{4c_1^2c_2+4c_1^2+6c_1c_2+4c_1-1+2c_2}{(1+c_2)(c_1+1)(c_1+c_1c_2+c_2)} - \frac{1}{4}\frac{2c_1+2c_1c_2+c_2}{(1+c_2)(c_2+c_2c_1+c_1)}$$
$$= -\frac{1}{8(c_2+1)(c_1+1)(c_1+c_1c_2+c_2)}<0.$$

2. $$\phi_{1\_SMSP}-\phi_{1\_MSP} = \frac{2c_1+4c_1c_2+1+4c_2}{(4c_1^2c_2+4c_1^2+6c_1c_2+4c_1+2c_2-1)} - \frac{c_2}{2c_1+2c_1c_2+c_2}$$
$$= \frac{2(c_2+1)(2c_1+1)(c_1+c_1c_2+c_2)}{(4c_1^2c_2+4c_1^2+6c_1c_2+4c_1-1+2c_2)(2c_1+2c_1c_2+c_2)}>0 \text{ (for } c_1>0.5),$$

$$\phi_{2\_SMSP} - \phi_{2\_MSP} = \frac{2c_1 - 1}{2c_1 + 4c_1c_2 + 1 + 4c_2} - \frac{c_1}{2c_2 + 2c_1c_2 + c_1)}$$
$$= -2\frac{c_1 + c_1c_2 + c_2}{(2c_1 + 4c_1c_2 + 1 + 4c_2)(2c_2 + 2c_1c_2 + c_1)} < 0.$$

3. $$\Pi_{1\_SMSP} - \Pi_{1\_MSP} = \frac{1}{64}\frac{(2c_1 + 4c_1c_2 + 1 + 4c_2)^2}{(1 + c_2)(c_1 + 1)^2(c_1 + c_1c_2 + c_2)} - \frac{1}{16}(2c_2 + 1) \times \frac{2c_2 + 2c_1c_2 + c_1}{(1 + c_2)(1 + c_1)(c_1 + c_1c_2 + c_2)} = \frac{1}{64(c_2 + 1)(c_1 + 1)^2(c_1 + c_1c_2 + c_2)} > 0.$$

$$\Pi_{2\_SMSP} - \Pi_{2\_MSP} = (2c_1 + 1)\left[\frac{4c_1^2c_2 + 4c_1^2 + 6c_1c_2 + 4c_1 - 1 + 2c_2}{32(1 + c_2)(c_1 + 1)^2(c_1 + c_1c_2 + c_2)} - \frac{2c_1 + 2c_1c_2 + c_2}{16(1 + c_2)(1 + c_1)(c_1 + c_1c_2 + c_2)}\right]$$
$$= -\frac{1}{32}\frac{2c_1 + 1}{(c_2 + 1)(c_1 + 1)^2(c_1 + c_1c_2 + c_2)} < 0.$$

4. Total quantity difference:

$$\frac{1}{8}\frac{c_2}{(c_2 + 1)(c_1 + 1)(c_1 + c_1c_2 + c_2)} > 0.$$

5. Total profits difference:

$$\frac{1}{64}\frac{1 + 4c_1}{(c_2 + 1)(c_1 + 1)^2(c_1 + c_1c_2 + c_2)} < 0.$$

Proof of Proposition 8:
If c is less than 0.5, the outcome is the same as the outcome when firm 1 is the Stackelberg leader in a standard quantity competition. The above inequalities trivially hold.

In the case where $c > 0.5$, simple comparisons prove the result.

The symmetric case is characterized by:

$$\phi_1 = \frac{6c + 4c^2 + 1}{4c^3 + 10c^2 + 6c - 1}, \quad q_1 = \frac{1}{8}\frac{6c + 4c^2 + 1}{c(3c + c^2 + 2)}, \quad \Pi_1 = \frac{1}{64}\frac{12c + 16c^4 + 48c^3 + 44c^2 + 1}{c(7c + 9c^2 + 5c^3 + c^4 + 2)},$$
$$\phi_2 = \frac{-1 + 2c}{6c + 4c^2 + 1}, \quad q_2 = \frac{1}{8}\frac{4c^3 + 10c^2 + 6c - 1}{c(4c^2 + c^3 + 5c + 2)}, \quad \Pi_2 = \frac{1}{32}\frac{-1 + 22c^2 + 24c^3 + 4c + 8c^4}{c(7c + 9c^2 + 5c^3 + c^4 + 2)}.$$

Recall that the equilibrium of the symmetric cost managers selection game is characterized by:

$$\phi_i = \frac{1}{3 + 2c} \quad q_i = \frac{1}{4c^2 + 3c + 2}\frac{3 + 2c}{} \quad \Pi_i = \frac{1}{64}\frac{4c^2 + 8c + 3}{4c^2 + c^3 + 5c + 2}$$

Checking the above inequalities is straightforward. □

Proof of Proposition 10:
The equilibrium quantity, price, and profit for each firm in the standard price competition case is:

$$q_0 = \frac{zc + 1 - c}{2 - z} \quad p_0 = \frac{1 + c}{2 - z} \quad \Pi_0 = \frac{(zc + 1 - c)^2}{(z - 2)^2}$$

The difference in profits is therefore:

$$\Pi_{MSP} - \Pi_0 = \frac{1}{16}(z-2)(2+z)\frac{(zc+1-c)^2}{z-1} - \frac{(zc+1-c)^2}{(z-2)^2}$$
$$= \frac{1}{16}(zc-c+1)^2 z^3 \frac{z-4}{(z-1)(z-2)^2}.$$

This has the same sign as z.

The difference in the prices is:

$$p_{MSP} - p_0 = \frac{1z^2c+zc+z-2c-2}{4\quad\quad z-1} - \frac{1+c}{2-z} = \frac{1}{4}z^2\frac{zc-c+1}{(z-1)(z-2)}.$$

The difference in the quantities is:

$$q_{MSP} - q_0 = \frac{1}{4}(2+z)(zc+1-c) - \frac{zc+1-c}{2-z} = \frac{1}{4}z^2\frac{zc-c+1}{z-2}.$$

Hence prices increase and quantities decrease.

□

Proof of Proposition 11:

The inverse demand function are given by

$$p_i = \frac{1+z}{1-z^2} - \left(\frac{1}{1-z^2}\right)q_i - \frac{z}{1-z^2}q_j.$$

We begin with the second stage. A manager for firm 1 of type ϕ_1 solves the following problem:

$$\max_{q_1}\left(\frac{1+z}{1-z^2} - \left(\frac{1}{1-z^2}\right)q_1 - \frac{z}{1-z^2}q_2\right)q_1 - \phi_1 q_2\left(\frac{1+z}{1-z^2} - \left(\frac{1}{1-z^2}\right)q_2 - \frac{z}{1-z^2}q_1\right).$$

The reaction function is given by

$$q_1 = \frac{1}{2} + \frac{1}{2}z - \frac{1}{2}zq_2 + \frac{1}{2}\phi_1 q_2 z.$$

Calculating the reaction function for the manager of firm 2 and solving for the quantities yield:

$$q_i(\phi_1,\phi_2) = -(1+z)\frac{-z+z\phi_i+2}{-4+z^2-\phi_1 z^2 - z^2\phi_2 + z^2\phi_2\phi_1}.$$

Owner 1's maximization problem is:

$$\max_{\phi_1\in[-1,1]}\left(\frac{1+z}{1-z^2} - \left(\frac{1}{1-z^2}\right)q_1(\phi_1,\phi_2)\left\{ - \frac{z}{1-z^2}q_2(\phi_1-\phi_2)\right)q_1(\phi_1,\phi_2).$$

With the reaction function

$$\phi_1 = (z\phi_2 - 2\phi_2 + 2 - z)\frac{z}{z^2\phi_2 + 2z\phi_2 - z^2 - 2z + 4}.$$

Solving for the reaction function of the second owner we get

$$\phi_2 = (z\phi_1 - 2\phi_1 + 2 - z)\frac{z}{z^2\phi_1 + 2z\phi_1 - z^2 - 2z + 4}.$$

With the final solution of:

$$\phi_1 = \phi_2 = \frac{z}{2+z}.$$

Substituting back we get the desired expressions for q_i, p_i. □

NOTES

1. Throughout the paper we will refer to managers with the pronoun 'he' and to owners with 'she'. The enraged reader is welcome to switch the genders of owners and managers.
2. Proofs of the results in the remainder of this section are relegated to the appendix.
3. As before, similar results hold for general linear demand and different marginal production costs.

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